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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

NATURAL PERIODS OF UNIFORM CANTILEVER BEAMS

By Lydik S. Jacobsen,[1] Esq.

Synopsis

An attempt has been made to give a quantitative answer to the question of the relative influence of different types of distortions on the natural periods of uniform cantilever beams elastically fixed at their bases. Very few buildings approximate the uniform and symmetrical beam dealt with in the theory, so that close agreement with actuality cannot be expected if the theory is applied to buildings. However, it is felt that the study enables the designer of earthquake resistant structures to put his finger on the most important factors entering into a determination of periods. The order or magnitude of errors committed by neglecting either flexure or shear in the building and by neglecting elastic yielding of the ground may be inferred from the tables, curves, and approximate formulas of the paper. Quantitative data relating to ground rigidities are as yet rather speculative, but it is hoped that the theory presented will help investigators to form reasonable estimates of the magnitudes of the ground rigidities from field data of natural vibrational periods of buildings.

Introduction

Transverse vibration frequencies of uniform beams have usually been computed from a consideration of the dynamic, flexural distortions. (The term "flexural distortion" refers to distortions produced by bending moments only.) When the beams are long and slender, experimental evidence is in fair agreement with the theoretical results, especially for the fundamental mode, but if higher modes are considered, or if the beams are relatively stubby, the deviations from the simple theory increase with the order of the mode. Retaining Lord Rayleigh's correction for the effect of rotatory inertia of the beams, S. Timoshenko[2] introduced a second correction for shear and solved the problem of frequencies for a uniformly loaded beam resting simply on rigid supports. In

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Note.—Written comments are invited for immediate publication; to ensure publication, the last discussion should be submitted by **July 15, 1938.**

[1] Prof. of Mech. and Civ. Eng., Stanford Univ., Stanford University, Calif.

[2] "Vibration Problems in Engineering," by S. Timoshenko, 1928, p. 231.

the case of a beam of rectangular cross-section, he found that the shear correction is four times larger than the one due to rotatory inertia. Using methods of successive approximations, V. I. Novotorzev[3] studied the effect of shear, rotatory inertia, and longitudinal elasticity on the fundamental frequency of beams and of simple bents. He found that for a rigidly built-in cantilever beam of rectangular cross-section, with a length to depth ratio of 4.56, the effect of shear distortion is to lower the frequency 3.3% from that obtained by considering flexural distortions alone, and that the combined effect of flexure, shear, and rotatory inertia amounted to a 4.2% lowering of the frequency. (The term, "shear distortion," refers to distortions produced by shear only.)

In regard to the higher mode frequencies the correction occasioned by shear will increase with the order of the mode until very soon a much better approximation of the frequencies results from a consideration of shear alone rather than from flexure alone. Quantitative data for the higher modes of cantilever beams, to the writer's knowledge, have not been previously published.

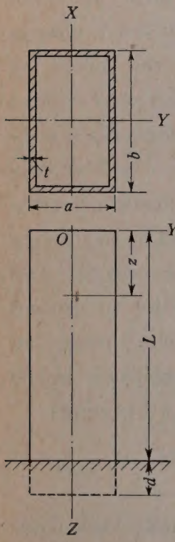
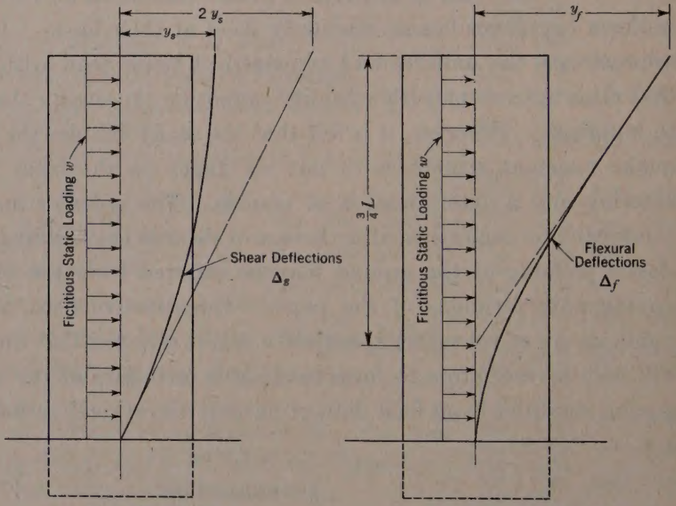


FIG. 1.—CO-ORDINATE SYSTEM AND DIMENSIONS


$2y_s$
y_s
Fictitious Static Loading w
Shear Deflections Δ_s
y_f
$\frac{3}{4}L$
Fictitious Static Loading w
Flexural Deflections Δ_f

(a) DUE TO SHEAR

(b) DUE TO FLEXURE

FIG. 2.—STATIC DEFLECTIONS OF A UNIFORMLY LOADED CANTILEVER

In vibration studies of beams it has been customary to assume simple end conditions as, for instance: (1) Entirely free; (2) rigid for translation and free for rotation; (3) rigid for rotation and free for translation; and (4) rigidly built-in. Frequency studies of cantilever beams with elastic constraints at the built-in end have undoubtedly been made for cases of flexure alone and shear alone,[4] but it is believed that the cantilever frequencies due to flexure, shear, translatory and rotatory elastic yielding of the built-in end have not been published before.

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[3] *Bulletin No. 70*, Academie des Sciences de l'Union des Republiques Sovietiques Socialistes, 1935.

[4] *Zeitschrift für technische Physik*, No. 7, 1927, p. 269. (E. Schwerin considers elastic restraints at the end of a non-uniform beam subjected to flexure only.)

Consider a vertical, uniform cantilever beam of rectangular cross-section and of a free length L. The beam is "planted" to a depth, d, in an elastic medium (see Fig. 1).

Notation.—The symbols used in this paper are defined where they first appear and are assembled for reference in Appendix I.

STATIC DISTORTIONS OF THE BEAM ITSELF

Since the elastic properties of the cantilever beam have something to do with its vibrations, the first step is to examine some of the static distortions that would occur if a hypothetical, horizontal, uniform loading of the same intensity as the uniform weight of the beam were to act in the Y direction. The beam will then deflect horizontally so that shear deflections, Δ_s, and flexural deflections, Δ_f, become established as shown in Fig. 2. If the origin of the co-ordinate system is at the top of the beam and z is measured positive downward, the deflections of a cantilever beam with a rigidly built-in end are:

$$\Delta_s = \frac{Y_s\, w}{2\, A\, G}\,(L^2 - z^2) \quad \text{(1a)}$$

$$(\Delta_s)_{\max} = y_s = \frac{Y_s\, w\, L^2}{2\, A\, G} \quad \text{(1b)}$$

$$\Delta_f = \frac{w}{24\, E\, I_x}\,(3\, L^4 - 4\, L^3\, z + z^4) \quad \text{(1c)}$$

$$(\Delta_f)_{\max} = y_f = \frac{w\, L^4}{8\, E\, I_x} \quad \text{(1d)}$$

in which, w is the weight per unit height of beam, in pounds per inch; A is the cross-sectional area of beam, in square inches; I_x is the moment of inertia of beam section about the X-axis, in inches^4; E is the modulus of elasticity, in pounds per square inch; G is the modulus of elasticity in shear, in pounds per square inch; and, Y_s is the shear deflection coefficient, which is dimensionless.

The value of Y_s is generally not the same as the shearing stress distribution coefficient of 1.5 resulting from the simplified theory of a parabolic shearing stress distribution over the cross-section. According to Theodor von Kármán, M. Am. Soc. C. E., and F. Seewald[5] the influences of the non-linear distribution of longitudinal stresses as well as the transverse stresses due to the lateral deformation depend on the value of Poisson's ratio, $\frac{1}{m}$, and for a solid rectangular cross-section of a narrow width must be taken into account as follows:

$$Y_s = \frac{3}{2} - \frac{3\, m}{10\,(m+1)} - \frac{3}{4\,(m+1)} \quad \text{(2)}$$

The first member on the right-hand side of Equation (2) is the usual shear distribution coefficient first evaluated by Grasshof.[6] The second member gives the correction due to the longitudinal stresses, and the third represents the correction necessitated by the lateral deformation of the beam. Thus, if $\frac{1}{m}$ is one-third, $Y_s = 1.500 - 0.225 - 0.188 = 1.087$; and, if $\frac{1}{m}$ is one-fourth, $Y_s = 1.50 - 0.24 - 0.15 = 1.11$.

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[5] Abhandlungen aus dem Aerodynamischen Institut der Techn. Hochschule, Aachen, Vol. 7, pp. 3, 11, Berlin, 1927.

[6] "Der Stahlbau," by R. Gran Olsson, 1934, Vol. 2.

It should be mentioned that for any built-in beam, such as a cantilever, several possible but different assumptions relating to the type of distortional constraint, as amplified by warping of the cross-section at the built-in fixed end, lead to different expressions for the effect of shearing force on the deflection.[7]

If the cross-section of the beam is different from the rectangular (as, for instance, those shown in Fig. 3) the foregoing considerations in regard to the value of Y_s do not apply quantitatively, and the designer is forced to assign somewhat arbitrary values to the shear deflection coefficients. In view of the fact that deflection experiments on H-beams in the Y-direction, Fig. 3(c), are in fair agreement with the simple theory when Y_s is taken equal to the shearing stress distribution coefficient, this assumption may be accepted until better approximations become available. The simple static moment formula of elementary strength of materials, therefore, is used,

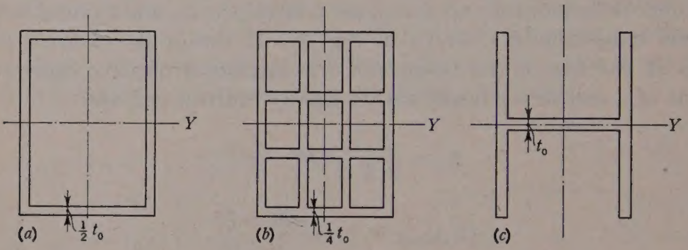

FIG. 3.—EXAMPLES OF CROSS-SECTIONS

$$Y_s = \frac{A \int_0^{a/2} y \, dA}{t_0 \, I_x} \qquad (3)$$

Example 1.—If t_0 (the thickness of the section at the neutral axis) is small in comparison with a and b, and if $a = b$, the value of Y_s for the box section is 2.25; for the cellular section, 2.58; and for the H-section, 3.11. These values are two to three times greater than for the rectangular solid cross-section; in other words, shear is of relatively great importance in a study of the sections shown in Fig. 3.

In addition to the fictitious horizontal loadings producing the lateral deflections in the Y and in the X-directions, the beam may be twisted about its vertical axis by a fictitious torsional twisting moment of $w\,k$ lb-in. per unit height. Under these circumstances the angle of twist will be given by:

$$\phi = \frac{w\,k}{2\,G\,R_t}(L^2 - z^2) \qquad (4a)$$

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[7] "Theory of Elasticity," by S. Timoshenko, McGraw-Hill, 1934, pp. 37, 150.

and,

$$(\phi)_{\max} = \phi_R = \frac{w\,k\,L^2}{2\,G\,R_t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4b)$$

in which k = radius of gyration of the cross-section, in inches; and $G\,R_t$ = torsional rigidity, in pound-inches². The value of R_t for the beam of rectangular cross-section is found from the St. Venant table printed in most textbooks.[8] If the cross-section of the beam is of the types shown in Fig. 3, R_t is best evaluated by use of the Prandtl membrane analogy, also found in textbooks.[8] However, if the box section or the cellular section has transverse horizontal diaphragms spaced at regular vertical intervals, as in the case of buildings with floors, the torsional shear distribution is influenced by these diaphragms, and somewhat arbitrary assumptions must be made in regard to the torsional properties of the beam. It is suggested that in this case R_t be defined as a summation of the products of the individual areas by the squares of their normal distances from the center of twist. For example, in the case of the rectangular box section of thickness, t,

$$R_t = \frac{1}{2}\,(a\,b^2 + a^2\,b)\,t\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5a)$$

If the membrane analogy had been used:

$$R_t = 2\,\frac{a^2\,b^2}{a+b}\,t\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5b)$$

Equations (5*a*) and (5*b*) give identical results when $b = a$, but when $b = 2\,a$, the first, and recommended, expression gives a value of 12% greater rigidity than the second. The arbitrary assumptions are especially necessary when sections of the cellular type are considered.

HYPOTHETICALLY ISOLATED VIBRATION FREQUENCIES OF A RIGIDLY BUILT-IN CANTILEVER BEAM

The transverse vibrations resulting from a consideration of shear alone are found from the well-known equation of motion:[9]

$$\frac{\partial^2 y}{\partial z^2} = \alpha^2\,\frac{\partial^2 y}{\partial t^2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

in which $\alpha^2 = \frac{Y_s\,w}{g\,A\,G}$, giving the frequency equation:

$$\cos \alpha\,p\,L = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

which means that the hypothetically isolated periods due to shear only may be expressed by:

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[8] See, for instance, "Strength of Materials," by S. Timoshenko, Van Nostrand, 1930, Vol. I, p. 77, and Vol. II, p. 623.

[9] "Vibration Problems in Engineering," by S. Timoshenko, p. 216.

$$(T_s)_1 = 0.288\sqrt{y_s} \qquad \text{(8a)}$$

$$(T_s)_2 = \frac{1}{3}(T_s)_1 \qquad \text{(8b)}$$

$$(T_s)_3 = \frac{1}{5}(T_s)_1 \qquad \text{(8c)}$$

$$(T_s)_4 = \frac{1}{7}(T_s)_1 \qquad \text{(8d)}$$

in which the maximum shear deflection, y_s, is given in inches. A consideration of flexural distortions only leads to the equally well-known equation of motion[10]:

$$\frac{\partial^4 y}{\partial z^4} = -\beta^2 \frac{\partial^2 y}{\partial t^2} \qquad \text{(9)}$$

in which $\beta^2 = \frac{w}{g I_x E}$, giving a frequency equation:

$$\cos\sqrt{\beta\, p\, L^2}\cosh\sqrt{\beta\, p\, L^2} = -1 \qquad \text{(10)}$$

which means that the hypothetically isolated periods due to flexure only may be expressed by:

$$(T_f)_1 = 0.258\sqrt{y_f} \qquad \text{(11a)}$$

$$(T_f)_2 = \frac{1}{6.27}(T_f)_1 \qquad \text{(11b)}$$

$$(T_f)_3 = \frac{1}{17.6}(T_f)_1 \qquad \text{(11c)}$$

$$(T_f)_4 = \frac{1}{34.4}(T_f)_1 \qquad \text{(11d)}$$

in which the maximum flexural deflection, y_f, is given in inches.

The torsional vibrations are not influenced by flexure since they involve shear or torsional distortions only. They may be obtained from Equations (6), (7), and (8), therefore, by replacing the coefficient, α, with α_ϕ, giving:

$$\frac{\partial^2 \phi}{\partial z^2} = \alpha_\phi^2 \frac{\partial^2 \phi}{\partial t^2} \qquad \text{(12)}$$

in which $\alpha_\phi^2 = \frac{w\, k^2}{g\, R_t\, G}$, giving a frequency equation:

$$\cos \alpha_\phi\, p\, L = 0 \qquad \text{(13)}$$

Therefore, the actual torsional periods are expressed as follows:

$$(T_R)_1 = 0.288\sqrt{k\,\phi_R} \qquad \text{(14a)}$$

$$(T_R)_2 = \frac{1}{3}(T_R)_1 \qquad \text{(14b)}$$

$$(T_R)_3 = \frac{1}{5}(T_R)_1 \qquad \text{(14c)}$$

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[10] "Vibration Problems in Engineering," by S. Timoshenko, p. 222.

and,

$$(T_R)_4 = \frac{1}{7}(T_R)_1 \ldots\ldots\ldots\ldots (14d)$$

in which the maximum torsional angle at the top of the beam is ϕ_R.

TRANSVERSE PERIODS OF A RIGIDLY BUILT-IN CANTILEVER BEAM

Before considering the elastically yielding end fastening of the beam, it is interesting to inquire as to the extent to which flexural and shear distortions are of importance in a study of the transverse periods of the rigidly built-in beam. For very stubby beams Equations (6), (7), and (8) give good results, and for very slender beams Equations (9), (10), and (11) apply. If the beams are neither stubby nor slender, Equation (45a) of Appendix II must be used. Table 3(a) (Appendix II), gives solutions of this equation in the form of period ratios, $\frac{T}{T_s}$ and $\frac{T}{T_f}$, in terms of the flexure-shear ratio, $\mu = \frac{y_f}{y_s}$. Thus, if the maximum static deflections from Equations (1) of the fictitiously loaded beam are equal, the ratio, μ, is unity, and the true periods will be:

For the fundamental mode,

$$T_1 = (1.32)\ 0.288\ \sqrt{y_s}, \quad \text{or} \quad T_1 = (1.47)\ 0.258\ \sqrt{y_f};$$

for the second mode,

$$T_2 = (1.20)\ \frac{0.288}{3}\ \sqrt{y_s}, \quad \text{or} \quad T_2 = (2.80)\ \frac{0.258}{6.27}\ \sqrt{y_f};$$

and for the fourth-mode period,

$$T_4 = (1.03)\ \frac{0.288}{7}\ \sqrt{y_s}, \quad \text{or} \quad T_4 = (5.64)\ \frac{0.258}{34.4}\ \sqrt{y_f}.$$

A comparison of the ratios, $\frac{T}{T_s}$ and $\frac{T}{T_f}$, of Table 3(a) (Appendix II) for any specific value of μ and for a specific mode of vibration gives an idea of the relative importance of shearing and of the flexural distortions of the beam in regard to period calculations. Thus, for the fundamental mode of the beam with $\mu = 1$, the ratios, $\frac{T}{T_s} = 1.32$, and $\frac{T}{T_f} = 1.47$, signify that if flexure is neglected entirely and if only shear distortions are considered, the true period is 32% longer than the computed period. On the other hand, if shear is neglected the true period is 47% longer than the period resulting from a consideration of flexure only. Therefore, in this case, the error due to neglecting shear is greater than that resulting from a neglect of flexure. When $\mu = 1.25$, the errors in the period due to neglecting either shear or flexure are equal and amount to 40 per cent. For values of μ greater than 1.25 a better approximation of the fundamental period results from a consideration of flexure only than from one of shear only.

For the second-mode periods the errors due to neglecting either shear or flexure are not equal until the ratio, μ, is 5.4. At this value the errors are 56 per cent. In the third-mode and the fourth-mode periods equalities of errors for the two methods of computation do not occur within the limits of extrapolation of Table 3(*a*) (Appendix II). In general, one may conclude that for cantilever beams, rigidly built-in, a consideration of shearing distortions rather than one of flexural distortions is of importance for the higher modes of vibration unless the beam is very slender so that the ratio, μ, becomes large.

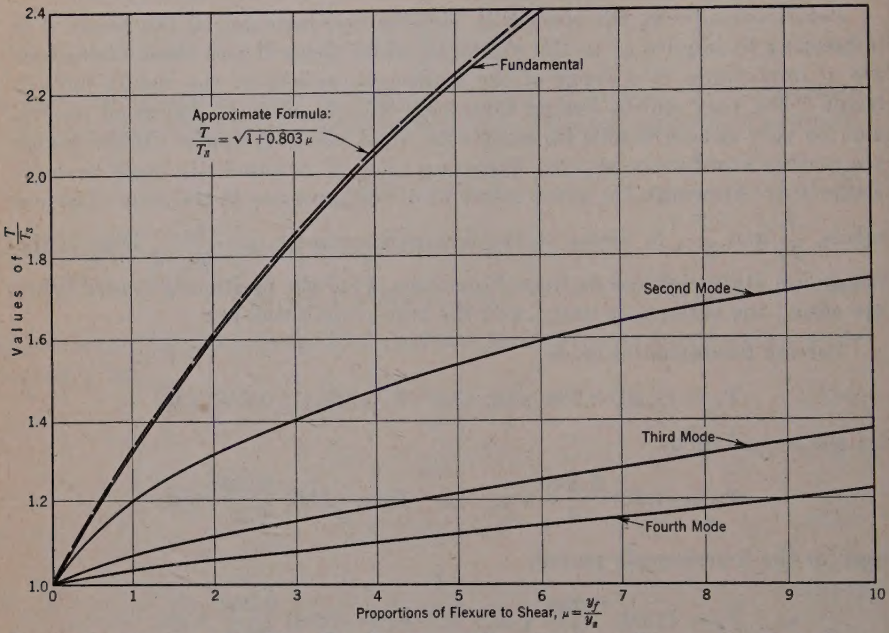

FIG. 4.—GENERAL CURVES FOR FIRST FOUR TRANSVERSE MODE PERIODS OF UNIFORM CANTILEVER BEAMS SUBJECTED TO DYNAMIC SHEAR AS WELL AS TO DYNAMIC FLEXURE; RIGID GROUND

In order to emphasize the significance of Table 3(*a*) (Appendix II), Fig. 4 and Tables 3(*b*), 3(*c*), and 3(*d*) have been included. Table 3(*b*) (Appendix II) refers to a cantilever beam with a solid square cross-section. The shear deflection coefficient, Y_s, is taken equal to 1.2, and the modulus of elasticity, E, is taken equal to 2.5 G. In this case equalities of errors in period of the fundamental mode occur when the height-to-depth ratio, $\frac{L}{a}$, is 1.1; for the second mode when $\frac{L}{a}$ is 2.3; for the third mode when $\frac{L}{a}$ is 3.9; and for the fourth mode when $\frac{L}{a}$ is 5.3. This means that in most beams of solid square cross-section, only relatively stubby ones require that both types of distortions be considered for computations of periods. Shear distortions may be neglected except for the higher modes of vibration.

It is interesting to note that for $\frac{L}{a} = 4$, Ratio $\frac{T}{T_f}$ is 1.04, which means that the lengthening of the period due to shear is 4 per cent. This is in good agreement with the result obtained by Novotorzev[3] of 3.3% for a beam with $\frac{L}{a} = 4.56$.

In 1894, S. Dunkerley[11] showed that the resultant period, T_r, of a complicated system may be expressed empirically to a fair degree of approximation in terms of the "isolated component" periods, T_a, T_b, $\cdots$ T_n, as follows:

$$T_r^2 = T_a^2 + T_b^2 + \cdots T_n^2 \qquad (15)$$

In the present case, in which the two "isolated component" periods are the one due to shear alone, and the one due to flexure alone,

$$T_{sf}^2 = T_s^2 + T_f^2 = 0.228^2\, y_s + 0.258^2\, y_f \qquad (16)$$

and,

$$\frac{T_{sf}}{T_s} = \sqrt{1 + 0.803\,\frac{y_f}{y_s}} = \sqrt{1 + 0.803\,\mu} \qquad (17)$$

The empirical formula, Equation (17), is in good agreement with the exact theory as may be seen from the dotted curve on Fig. 4. In this case the error is positive, but in other cases it may be either positive or negative (see Fig. 6, for instance). However, it should be emphasized that the approximate formula does not have the slightest application to the period ratios of the higher modes, and that it is precisely for these modes that considerations of flexure alone give erroneous results unless the beams are very slender indeed.

Table 3(*c*) (Appendix II) relates to a cantilever beam of a square box cross-section with Y_s equal to 2.25 and with E equal to 2.5 G. In this case equality of errors in periods for computations involving shear alone and flexure alone occurs for the fundamental mode when $\frac{L}{a}$ is 2.2; for the second mode when $\frac{L}{a}$ is 4.5; and for the third mode when $\frac{L}{a}$ is 7.6. Table 3(*c*) does not extend far enough to give this ratio for the fourth mode. It is seen that in the case of the cantilever beam with a square box cross-section shearing as well as flexural distortions must be considered even if the values of $\frac{L}{a}$ are appreciable.

The period ratios for the beam with the rectangular box cross-section of a width-to-depth ratio of 3.1 are given in Table 3(*d*) (Appendix II). In this case Y_s is 4.45 so that a relatively large shear effect is present. From Table 3(*d*) it is found that equalities of errors in periods for the two types of distortion occur when $\frac{L}{a}$ is 3.4 for the fundamental mode, when $\frac{L}{a}$ is 7.2 for the second mode, and when $\frac{L}{a}$ is between 12 and 13 for the third mode.

If the beam with rectangular box cross-section is considered to be somewhat akin to a very much idealized building, it is clear that the influence of shearing distortions on its natural periods is far from negligible; indeed, for buildings of ordinary proportions a consideration of shear alone will give better approximations of the true natural periods, including the second mode and those above it, than a consideration of flexure alone. Of course, for the fundamental-mode period both types of distortion are of importance.

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[11] *Philosophical Transactions*, Royal Soc., Lond., Vol. 185 (1894), p. 311.

Ratios of Natural Periods

Table 3(*d*) (Appendix II) also gives the ratios of the true fundamental periods to the true higher mode periods for the rigidly built-in cantilever beam with rectangular box cross-section of $\frac{b}{a} = 3.1$. The ratios of 3, 5, and 7 for the case of shear alone (Equations (8)) are obtained when $\frac{L}{a}$ approaches zero, and the ratios of 6.27, 17.6, and 34.4 for the case of flexure alone are theoretically approached as $\frac{L}{a}$ becomes infinite. When $\frac{L}{a}$ is 10—a very large ratio for a building—the period ratios, 5.28, 11.1, and 17.5, are quite far from either of the two sets of limiting ratios. Finally, it may be concluded that even in the case of a rigidly built-in, uniformly loaded, cantilever beam the ratios of natural periods are not very simple.

End Constraints of the Cantilever Beam

In proceeding to a study of the effect, on the periods, of the elastic constraint at the built-in end, it will be assumed that the medium in which the cantilever beam is "planted" is capable of exerting elastic restoring forces and moments on the end of the beam. The inertia of the medium is neglected and, for the time being, the restoring force properties of the medium and "planting" may be thought of as resulting from static experiments. Appendix III includes a description of a crude, rational method of relating the restoring properties of the ground and "planting" to ground deformation moduli.

The translatory rigidities offered by the ground to the "planted" beam in the X-direction and in the Y-direction are defined by R_x and R_y. The units are pounds per inch of translation. If the fictitious horizontal loading of the weight, $w\,L$, of the beam itself is assumed to act in the Y-direction, the corresponding horizontal translation of the base of the beam in this direction, in inches, is given by:

$$y_t = \frac{w\,L}{R_y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

In regard to the rotatory or tilting rigidity of the ground, it is assumed that the horizontal axis about which rotation or tilting occurs lies in the plane of the surface of the ground. The rotatory restraints of the ground upon the beam about the X- and Y-axes in the plane of the ground are then given by $R_{x\theta}$ and $R_{y\theta}$. The units are pound-inches per radian. If the fictitious horizontal loading of the weight of the beam itself, $w\,L$, is assumed to act in the Y-direction, the vertical axis of the beam will be inclined in the $Z\,Y$ plane and the maximum linear displacement at the top of the beam, in inches, will be given by,

$$y_r = \frac{w\,L^3}{2\,R_{y\theta}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19)$$

The fifth rigidity of the ground relates to the torsional constraint of the ground acting through the "planted" part of the beam. It will be assumed again that this rigidity is measured about the vertical axis of the beam in the plane of the ground. It is denoted by R_ϕ and its dimensions are pound-inches per radian. The effect of the fictitious torsional moment of $w\,L\,k$ lb-in. about the vertical axis of the beam will be an angle of rotation of the base of the beam, in radians, with respect to the ground of the magnitude,

$$\phi_g = \frac{w\,L\,k}{R_\phi} \qquad (20)$$

It is important to note that in the case of a homogeneous elastic ground the five rigidities, R_x, R_y, $R_{x\theta}$, $R_{y\theta}$, and R_ϕ are functions of the properties of the ground as well as of the dimensions, a, b, and d of the "planting."

INFLUENCE OF THE GROUND ON THE TORSIONAL PERIODS

Since flexure in the building as a whole does not affect its torsional properties, the torsional-mode periods of the elastically built-in cantilever beam can be found from a general solution of Equation (47*a*), of Appendix II, if the shear coefficients, α and τ, are replaced by the torsional coefficients, α_ϕ (in radians per inch per second) and $\tau\phi$ (in second² per inch per radian), defined as follows:

$$\alpha_\phi = \sqrt{\frac{w\,k^2}{g\,R_t\,G}} = \sqrt{\frac{2\,k\,\phi_R}{g\,L^2}} = 0.072\sqrt{\frac{k\,\phi_R}{L^2}} \qquad (21a)$$

and,

$$\tau_\phi = \frac{w\,k^2}{g\,R_\phi} = \frac{k\,\phi_g}{g\,L} \qquad (21b)$$

With these substitutions, Equation (47*a*) (Appendix II) may be written thus:

$$\frac{0.454\,\sqrt{k\,\phi_R}}{T}\tan\left(\frac{0.454\,\sqrt{k\,\phi_R}}{T}\right) = 2\,\frac{\phi_R}{\phi_g} \qquad (22)$$

Solutions of this transcendental equation yield the torsional periods for any cantilever beam with elastic, torsional constraint at the built-in end for assumed values of the torsional rigidity ratio, $\psi = \frac{\phi_g}{\phi_R}$. The torsional periods of the nth mode of vibration are expressed by,

$$T_n = \left(\frac{T_{Rg}}{T_R}\right)_n \frac{0.288}{2\,n - 1}\sqrt{k\,\phi_R} \qquad (23)$$

Fig. 5 shows the values of the period ratios that must be used in Equation (23). Furthermore, it shows that as elastic torsional yielding of the ground increases, the fundamental torsional period becomes longer and longer, so that the second-mode torsional period approaches the fundamental mode torsional period for the beam with both ends free; similarly, the third-mode period approaches the second-mode period for the free beam, etc. The ratios of the fundamental torsional periods to those of the higher modes are close to the values 3, 5, 7, and 9 for small values of the ratio, ψ.

If Dunkerley's empirical formula (see Equation (15)) is applied to the fundamental mode of torsion,

$$T_{Rg} = \sqrt{T_R^2 + T_g^2} = \sqrt{(0.830\,\phi_R + 1.024\,\phi_g)\,k} \quad \ldots\ldots\ldots (24)$$

and,

$$\frac{T_{Rg}}{T_R} = \sqrt{1 + 1.23\,\frac{\phi_G}{\phi_R}} = \sqrt{1 + 1.23\,\psi} \quad \ldots\ldots\ldots\ldots (25)$$

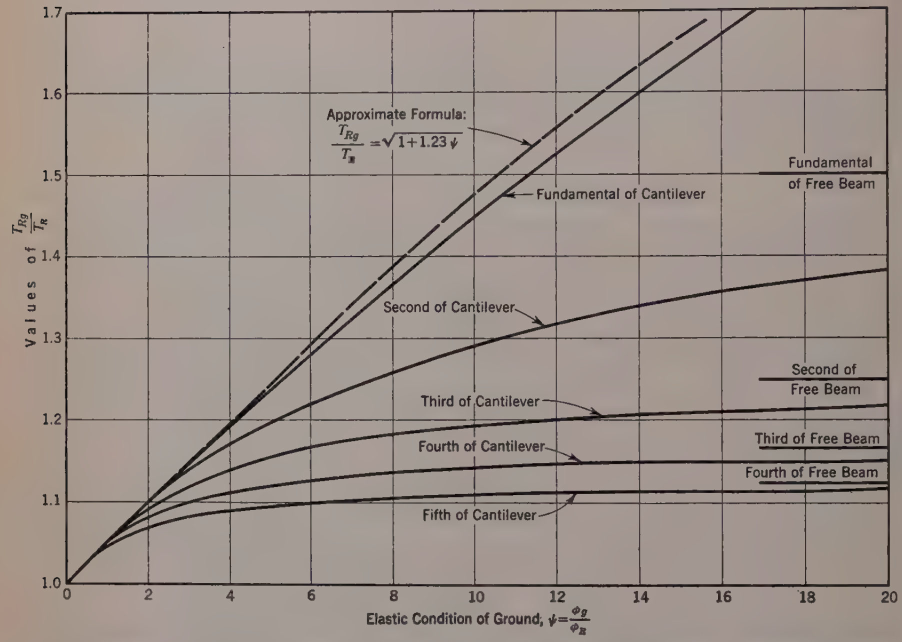

FIG. 5.—GENERAL CURVES FOR FIRST FIVE TORSIONAL MODE PERIODS OF UNIFORM CANTILEVER BEAMS "PLANTED" IN A TORSIONALLY ELASTIC GROUND

The results of Equation (25) are plotted on Fig. 5 as a broken-line curve. It is suggested that the approximate formula is accurate enough for practical purposes.

INFLUENCE OF THE GROUND ON THE PERIODS OF THE TRANSVERSE VIBRATIONS

In Appendix II the general problem of the free vibration periods of cantilever beams has been solved by considering shear and flexure of the beams as well as translatory and rotatory elastic yielding of the ground. Since the writer's primary interest in the application of the results centers on idealized buildings, a specific but fictitious building has been chosen to illustrate the significance of the theoretical deductions in Appendix II.

Example 2.—A cantilever beam of rectangular box cross-section, Fig. 3(*a*), is defined as follows: (1) Period due to shear alone, 1 sec = $(T_s)_1$; (2) height, 200 ft, or 2 400 in. = L; (3) depth, 52.5 ft, or 630 in. = a; (4) width, 170.0 ft, or 2 040 in. = b; (5) radius of gyration, 35 ft, or 420 in. = k; and (6) depth of "planting," 20 ft, or 240 in. = d.

The ground in which the cantilever beam is planted is defined by: (7) Horizontal deformation modulus, ϵ_h, and shear deformation modulus, ϵ_s, are equal (see Appendix III); and, (8) vertical deformation modulus, ϵ_v, is twenty-five times larger than ϵ_h; or $\frac{\epsilon_v}{\epsilon_h} = \gamma = 25$.

The yielding of the ground in translation, in rotation, and in shear may then be expressed, according to Appendix III, by the ground constants, ϵ_h and γ, taken in the proper combination with the physical dimensions of the building. The ratio of the translatory ground deflection, y_t, to the maximum shear deflection, y_s, of the beam is then sufficient for characterizing uniquely the yielding of the ground. This ratio is called ν $\left(\text{that is, } \nu = \frac{y_t}{y_s}\right)$.

The discussion of the chosen example is confined to a study of the transverse translatory vibration periods in the Y-direction and to the torsional vibration periods about the Z-axis.

By considering the specifications of the beam and some of the given equations it is found that: $y_s = 12.1$ in. (Equation (8a)); $Y_{sx} = 4.45$ and $Y_{sy} = 1.70$ (Equation (3)); $A = 2\,(a + b)\,t = 5\,340\,t$ sq in.; $I_x = (\frac{1}{2}\,a^2\,b + \frac{1}{6}\,a^3)\,t = 444 \times 10^6\,t$ in.^{4}; $\frac{w}{G} = 5.04 \times 10^{-3}\,t$ in. (Equation (1b)); $\frac{w}{E} = 2.02 \times 10^{-3}\,t$ in. (if $E = 2.5\,G$); $y_f = 18.6$ in. (Equation (1d)); $(T_f)_1 = 1.11$ sec (Equation (11a)); $\mu = \frac{y_f}{y_s} = 1.54$; $R_t = 1.71 \times 10^9\,t$ in.^{4} (Equations (5)); $\phi_R = 3.57 \times 10^{-3}$ radians (Equation (4b)); and, $(T_R)_1 = 0.353$ sec (Equation (14a)).

From the specifications of the ground and from Equation (53a) (Appendix III): $\frac{y_t}{y_s} = \nu$ (different values of ν to be assumed); $\frac{y_r}{y_t} = \frac{(12\,d + 6\,b)\,L^2}{\gamma\,a^3 + 8\,d^3} = 13.8$; $\lambda = \frac{y_r}{y_s} = \frac{y_r}{y_t}\,\frac{y_t}{y_s} = 13.8\,\nu$; and, $\psi = 3.86\,\nu$ from Equation (54) (Appendix III).

Torsional Periods in Example 2.—The torsional coefficients, $\frac{T_{Rg}}{T_R}$, of Equation (23) are easily found from Fig. 5 for the values of ψ corresponding to the assumed values of ν. From the results, as given in Table 1, it is seen that the fundamental torsional period increases indefinitely as ν increases. This means that a cantilever loses its fundamental mode as ν approaches infinity, or as its constraint at the built-in end approaches zero. The second-mode period of the motion of the cantilever with built-in end changes into the fundamental-mode period of the beam with free ends as ν approaches infinity. The third-mode period becomes the second-mode period, etc.

Transverse Periods in Example 2.—The solution of the frequency Equation (44) from Appendix III for the structure described by the foregoing constants and for assumed values of the ratio, ν, is a long and laborious task. The results are plotted in Figs. 6, 7, and 8.

TABLE 1.—TORSIONAL PERIODS, IN SECONDS, FOR THE FIRST FIVE MODES OF VIBRATION OF THE CANTILEVER BEAM REPRESENTING THE IDEALIZED BUILDING CHOSEN AS AN ILLUSTRATIVE EXAMPLE OF THE THEORY

(The Elastic Condition of the End Fastening Is Measured Either by ν or by ψ. The Ratios of the Fundamental to the Higher Mode Periods Are Also Given.)

SYMBOLS	VALUES OF ν AND ψ									
ν	0	0.05	0.10	0.20	0.30	0.40	0.50	1.0	2.0	∞
ψ	0	0.193	0.386	0.770	1.16	1.54	1.93	3.86	7.72	∞
(*a*) FUNDAMENTAL TORSIONAL MODE										
T_1	0.351	0.386	0.417	0.474	0.530	0.579	0.628	0.806	1.17	∞
(*b*) SECOND TORSIONAL MODE										
T_2	0.117	0.128	0.136	0.146	0.153	0.158	0.162	0.166	0.170	0.176
$\frac{T_1}{T_2}$	3.00	3.02	3.07	3.29	3.47	3.66	3.88	4.86	6.88	....
(*c*) THIRD TORSIONAL MODE										
T_3	0.0702	0.0766	0.0795	0.0830	0.0844	0.0850	0.0857	0.0865	0.0870	0.0878
$\frac{T_1}{T_3}$	5.00	5.05	5.25	5.71	6.28	6.81	7.35	9.33	13.5	....
(*d*) FOURTH TORSIONAL MODE										
T_4	0.0502	0.0543	0.0558	0.0568	0.0573	0.0578	0.0583	0.0584	0.0585	0.0586
$\frac{T_1}{T_4}$	7.00	7.11	7.46	8.34	9.25	10.0	10.8	13.8	20.0	....
(*e*) FIFTH TORSIONAL MODE										
T_5	0.0390	0.0414	0.0425	0.0429	0.0432	0.0434	0.0436	0.0437	0.0437	0.0438
$\frac{T_1}{T_5}$	9.00	9.33	9.87	11.04	12.3	13.3	14.4	18.5	26.8	....

Referring to Table 2(*a*), Item No. 1 gives the transverse periods of the structure on a rigid ground, computed on the assumption that shear alone governs the periods; thus, for instance, the fourth-mode period becomes one-seventh of the fundamental period, or 0.143 sec. The periods recorded as Item No. 2 involve the assumption that flexure alone governs the vibration. Item No. 3, still relating to the condition of a rigid ground, gives the so-called true periods of the structure. Items Nos. 4 to 12, inclusive, give true periods of the structure on a yielding ground; the degree of yielding is expressed by the value of ν. Item No. 13, for $\nu = \infty$ (that is, for a beam free at both ends), is a limiting case, and, consequently, when applied to a building is a physical impossibility. Its value lies in the fact that it indicates how closely the results for $\nu = 2$ approach those for $\nu = \infty$. It should be noted that for the higher mode periods the agreement between Items Nos. 12 and 13 is fairly close. The fundamental-mode period, however, increases without limit, which means that instability of motion results. Item No. 14 gives the periods of the structure with both ends free, computed from shear distortions only, whereas Item No. 15 gives the periods resulting from considering flexure alone. The third column of Table 2(*a*), relating to the fundamental period, has been plotted in Fig. 6.

TABLE 2.—PERIODS AND PERIOD RATIOS FOR THE FIRST SIX MODES OF TRANSVERSE VIBRATION OF THE CANTILEVER BEAM REPRESENTING THE IDEALIZED BUILDING CHOSEN AS AN ILLUSTRATIVE EXAMPLE OF THE THEORY

Item No.*	Values of the ratio, ν	(*a*) PERIODS, IN SECONDS, FOR THE FOLLOWING MODES:						(*b*) PERIOD RATIOS				
		Fundamental	Second	Third	Fourth	Fifth	Sixth	$\frac{T_1}{T_2}$	$\frac{T_1}{T_3}$	$\frac{T_1}{T_4}$	$\frac{T_1}{T_5}$	$\frac{T_1}{T_6}$
1†	Rigid¶	1.00	0.333	0.200	0.143	0.111	0.0910	3.0	5.0	7.0	9.0	11.0
2‡	Rigid¶	1.11	0.176	0.0634	0.0323	0.0195	0.0130	6.27	17.6	34.4	56.9	85.0
3§	Rigid¶	1.48	0.425	0.216	0.149	0.114	0.0925	3.48	6.86	9.92	13.0	16.0
4	0.05	1.70	0.458	0.226	0.154	0.117	0.0957	3.71	7.52	11.1	14.5	17.8
5	0.10	1.90	0.472	0.233	0.157	0.119	0.0959	4.02	8.16	12.1	16.0	19.8
6	0.20	2.18	0.480	0.236	0.156	0.117	0.0888	5.22	9.23	14.0	18.6	24.6
7	0.30	2.56	0.472	0.232	0.140	0.108	0.0872	5.42	11.0	18.3	23.7	29.4
8	0.40	2.88	0.465	0.223	0.132	0.105	0.0867	6.20	12.9	21.8	27.4	33.2
9	0.50	3.11	0.452	0.210	0.131	0.104	0.0862	6.89	14.8	23.7	29.9	36.1
10	0.80	3.76	0.400	0.200	0.130	0.103	0.0857	10.3	18.8	28.9	36.5	43.9
11	1.0	4.10	0.367	0.196	0.130	0.103	0.0857	11.2	20.9	31.7	39.4	47.9
12	2.0	5.80	0.322	0.192	0.130	0.103	0.0857	18.0	30.2	44.6	56.3	67.7
13	∞	∞	0.320	0.192	0.130	0.103	0.0857	. . .	1.67**	2.46**	3.20**	3.73**
14†	∞	∞	0.250	0.167	0.125	0.100	0.0833	. . .	1.50**	2.00**	2.50**	3.00**
15‡	∞	∞	0.173	0.0628	0.0320	0.0194	0.0130	. . .	2.76**	5.50**	8.92**	13.3**

* With the exceptions noted, the types of distortion are those due to shear, flexure, translation, and rotation, combined.

† Shear distortion only.

‡ Flexure distortion only.

§ Shear distortion and flexure distortion.

¶ The elastic condition of the ground; the elastic condition of the end fastening is measured by the ratio, ν.

** In these items the numerator of the ratio is T_2 instead of T_1.

If Dunkerley's empirical formula (Equation (15)) is applied to the fundamental-mode period of the cantilever:

$$T_{sftr}^2 = 0.830\, y_s + 0.666\, y_f + 1.024\, y_t + 0.681\, y_r \ldots\ldots\ldots (26)$$

For the specific cantilever under consideration, in which: $\frac{y_f}{y_s} = 1.54$; $\frac{y_r}{y_s} = 13.8$; and, $\frac{y_t}{y_s} = \nu$: $T_{sftr}^2 = (1.855 + 10.42\,\nu)\, y_s$: and,

$$\frac{T_{sftr}}{T_s} = \sqrt{2.24 + 12.55\,\nu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The results of this empirical formula are given by the dotted curve on Fig. 6. It is seen that the error is negative in this case. However, it is suggested that for practical problems the Dunkerley formula, as expressed in the general form by Equations (8), is satisfactory.

Fig. 7 shows a plot of the higher mode periods as functions of ν, the yielding of the ground. The dotted, straight horizontal lines on the left and on the right of the individual graphs relate to the periods for zero and for infinite yielding of the ground. The second-mode period increases rather rapidly for a small yielding of the ground, reaches a maximum when ν is about 0.2, and then decreases along a smooth curve. When ν is 0.6 the period of the mode is the same as for the second mode with $\nu = 0$. As ν increases further, the period drops, and as ν approaches infinity the period approaches asymptotically the fundamental-mode period of 0.320 sec for the beam or structure with both ends free. A similar variation in period resulting from a yielding of the ground occurs for all the higher mode vibrations.

It has now been shown that the fundamental-mode period of the cantilever increases indefinitely as the yielding of the ground is increased; moreover, that the second-mode period of the cantilever on a rigid ground transforms into the fundamental-mode period of the beam with free ends as ν increases. The gradual transformations from the second mode to the fundamental, from the third to the second, from the fourth to the third, etc., are shown in Fig. 7. It is to be noted that the higher the mode, the smaller the value of ν necessary for bringing about the transformation.

On Fig. 8, the ratios of the fundamental to the higher mode periods are plotted as functions of ν. It is seen that these ratios are far from simple; in general, they increase with the yielding of the ground. If the curve for $\frac{T_1}{T_2}$ is considered, it is seen that the ratio of 6.3, due to flexure alone and for a rigid ground, really corresponds to a yielding ground of which the coefficient, ν, is more than 0.4. On the other hand, the ratio of 3 due to shear alone does not exist, since the curve for $\frac{T_1}{T_2}$ starts at 3.4 for a rigid ground.

The practical value of the tendencies indicated by these curves may be twofold: First, from observations of periods on existing buildings (the United States Coast and Geodetic Survey has done much observational work in California) the ground factors may be estimated. These factors must then be related to the geologic characteristics of the ground as well as to soil tests of foundation mechanics. Second, if enough types of cases have been studied, it becomes possible to predetermine the periods of proposed buildings with a fair degree of accuracy.

It will undoubtedly be realized that the significance of such analyses is not restricted to the somewhat narrow, as well as hypothetical, value of periods for purposes of aseismic design. The periods of the building proper are indices of its mass and elastic properties, but they cannot be evaluated correctly until the ground effect on the periods becomes separable. Finally, from the point of view of foundation mechanics the periods of existing buildings, when analyzed, will give information about the ground itself.

APPLICATIONS OF THE THEORY

The theory applies to any symmetrical cantilever beam provided that its weight per unit height, its shearing rigidity, and its flexural rigidity are constant. Whether or not the ground rigidities are given by the equations of Appendix II makes no difference as far as the application of the theory is

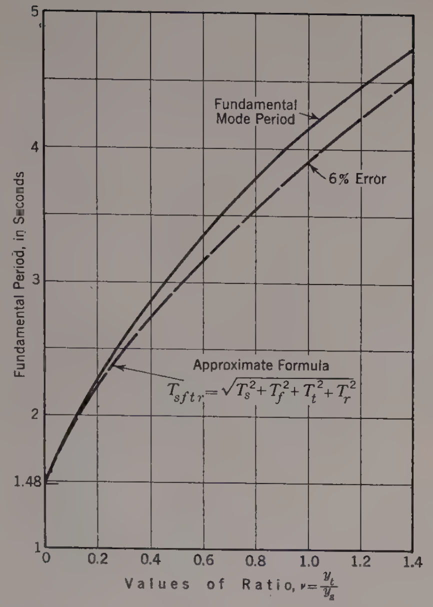

FIG. 6.—EFFECT OF AN ELASTICALLY YIELDING GROUND ON FUNDAMENTAL TRANSVERSE MODE PERIOD OF SPECIFIC CANTILEVER BEAM SUBJECTED TO SHEAR, FLEXURE, GROUND TRANSLATIONAL YIELDING AND GROUND ROTATIONAL YIELDING

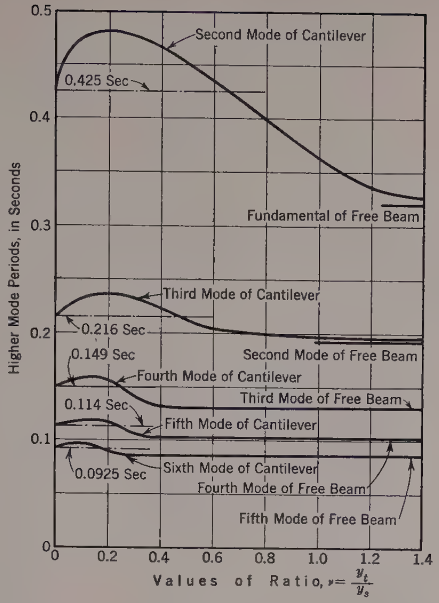

FIG. 7.—EFFECT OF ELASTIC GROUND ON HIGHER TRANSVERSE MODE PERIODS OF THE SPECIFIC CANTILEVER SUBJECTED TO THE FOUR TYPES OF YIELDING

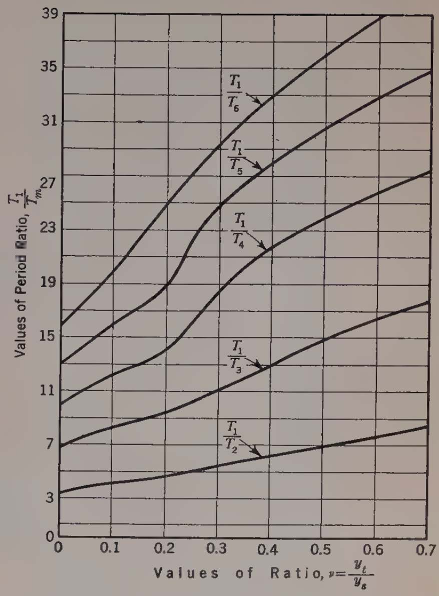

FIG. 8.—RATIOS OF FUNDAMENTAL TRANSVERSE MODE PERIOD TO HIGHER TRANSVERSE MODE PERIODS OF SPECIFIC CANTILEVER BEAM AS FUNCTIONS OF GROUND YIELDING

concerned. The necessary and sufficient condition for the ground is that its behavior must be elastic and symmetrical so that the ground coefficients, τ and ζ, Appendix II, can be evaluated.

In regard to buildings the theory will not apply with a satisfactory degree of quantitative accuracy on account of the fact that actual buildings, with a few exceptions, are far from satisfying the demands of constancy of mass and elastic properties with height; neither do they satisfy the condition of symmetry. It is thought, however, that qualitatively the theory explains the seemingly erratic data obtained from wind vibration records, or from shaking-machine vibration records of buildings. The theory indicates that for most buildings shear as well as flexure must be considered, and that the elastic yielding of the ground will influence the periods greatly.

If the data from period tests of actual symmetrical buildings are to be analyzed correctly, it becomes necessary to use graphical or mechanical methods for finding the component deflections due to shear, flexure, translatory, and rotatory rigidities of the ground.

A rough indication of the relative importance of the four factors influencing the periods of buildings may be obtained statistically from the observed data of the U. S. Coast and Geodetic Survey. The most important part of such an analysis is to choose carefully from the mass of data (about 200 to 300 buildings) the ones that satisfy most nearly the demands of twofold symmetry as well as isolation from other structures. Such a selection of data is found to decimate the number of buildings available.

Since it is known that the Dunkerley formula gives sufficiently accurate results for the fundamental-mode periods of uniform bars, and since, in the case of actual buildings, it may be supposed to apply, at least dimensionally, one may write:

$$T_{sftr} = \sqrt{c_1\,l + c_2\,l^2 + c_3\,l^3 + c_4\,l^4} \ldots\ldots\ldots\ldots\ldots (28)$$

in which l is a characteristic length-to-depth ratio of a building $\left(\text{that is, } \frac{L}{a} \text{ or } \frac{L}{b}\right)$, and the coefficients, c_1, c_2, c_3, and c_4, are to be determined from the available data. Thus, if four transverse fundamental periods of, say, two buildings of four different values of λ are known, four items of experimental information are available and may then determine uniquely the four coefficients. The significance of the four terms may then be explained as the component periods due to: Elastic ground translation, $\sqrt{c_1\,l}$; shear in the building, $\sqrt{c_2\,l^2}$; elastic ground tilting, $\sqrt{c_3\,l^3}$; and flexure in the building, $\sqrt{c_4\,l^4}$.

Of course, if more than four independent observations are available from which to find the four coefficients, the information will be redundant, and the method of least squares suggests itself for a statistical determination of the four coefficients.

GRAPHICAL AND MECHANICAL METHOD OF COMPUTING THE PERIODS

In its simplest form the problem of finding the period of one of the natural modes of vibration of an elastic system may be stated as the problem of finding a dynamic loading curve that will produce a geometrically similar, dynamic deflection curve. Although the two curves refer to dynamic conditions, they may be thought of as being static, so that a system subjected to an assumed dynamic loading curve may be treated by the well-known methods of statics in order to find its component deflections due to shear, flexure, and elastic ground yielding. The methods in question may be purely graphical, or graphical and numerical, or they may be a combination of static experimentation upon a mechanical system made up to represent the elastic properties of the system in shear, in flexure, and in end-fastening rigidity. It is possible, by means of simple strips, to build a static, elastic model of a building. The problem of finding the dynamic (that is, really, the static) loading curve that will produce an elastic deflection curve geometrically similar to the loading curve is merely a matter of sufficient experimentation by trial and error.

APPENDIX I

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering and Testing Materials[12] compiled by a committee of the American Standards Association, with Society representation, and approved by the Association in 1932:

x = horizontal co-ordinate;
y = horizontal co-ordinate; direction of transverse vibration;
z = vertical co-ordinate, origin at top of beam;
b = dimension of beam in horizontal direction;
a = dimension of beam in horizontal direction; direction of vibration;
L = dimension of beam in vertical direction;
A = cross-sectional area of beam;
Y_s = shear deflection coefficient for Y-direction;
I_x = moment of inertia of cross-section for vibrational direction;
w = weight per unit height of beam;
g = acceleration due to gravity;
E = modulus of elasticity for bending;
G = modulus of elasticity for shear;
m = Poisson's number;
k = radius of gyration of cross-section;
$G\,R_t$ = torsional rigidity of beam about vertical axis;
Δ_s = shear deflection due to fictitious gravity load in horizontal direction;
Δ_f = flexural deflection due to fictitious gravity load in horizontal direction;
ϕ = torsional angle due to fictitious moment, $w\,k$, per unit height;
y_s = shear deflection at top of beam; horizontal gravity loading;
y_f = flexural deflection at top of beam; horizontal gravity loading;

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[12] A.S.A.—*Z* 10*a*—1932.

μ = ratio, $\frac{y_f}{y_s}$;
ϕ_R = torsional angle at top of beam due to moment, $w\,k$, per unit height;
t = time in derivative notation; also, thickness of cellular sections;
t_0 = thickness of cross-section at neutral axis;
l = characteristic length ratio of beam or building;
c = coefficient;
d = depth of "planting" or embedment of beam in ground;
R_x = translatory rigidity of ground in X-direction;
R_y = translatory rigidity of ground in Y-direction;
$R_{x\theta}$ = rotatory or tilting rigidity of ground in $x\,z$-plane;
$R_{y\theta}$ = rotatory or tilting rigidity of ground in $y\,z$-plane;
R_g = torsional rigidity of ground for twist of beam about vertical axis;
y_t and x_t = displacements of top of building due to ground translational yielding;
y_r and x_r = displacements of top of building due to ground rotational yielding;
ϕ_g = angle of twist due to torsional yielding of the ground;
ϵ_v = deformation modulus of ground in vertical direction;
ϵ_h = deformation modulus of ground in horizontal direction;
ϵ_s = deformation modulus of ground in shear;
ν = ratio of $\frac{y_t}{y_s}$;
λ = ratio of $\frac{y_r}{y_s}$;
ψ = ratio of $\frac{\phi_g}{\phi_R}$;
α^2 = dynamic shearing constant of beam = $\frac{Y_s\,w}{g\,G\,A}\left(\frac{\text{seconds}^2}{\text{length}^2}\right)$;
β^2 = dynamic flexural constant of beam = $\frac{w}{g\,E\,I_x}\left(\frac{\text{seconds}^2}{\text{length}^4}\right)$;
p = time-frequency $\left(\frac{\text{radians}}{\text{second}}\right)$;
q = trigonometric space-frequency $\left(\frac{\text{radians}}{\text{length}}\right)$;
r = hyperbolic space-frequency $\left(\frac{\text{radians}}{\text{length}}\right)$;
u = space-frequency for case of flexure only = $\sqrt{\beta\,p}\left(\frac{\text{radians}}{\text{length}}\right)$;
ρ = beam parameter = $\frac{\beta^2}{\alpha^4}$ (seconds^{-2});
$f(z)$ = transverse loading intensity a function of z;
V = shear in beam;
M = moment in beam;
Z = normal space-function;
$f(t)$ = trigonometric function of time;
$A\ B\ C\ D$ = temporary constants of integration;
T = natural period of vibration (seconds);

T_1 = fundamental natural period;
T_n = nth mode natural period;
T_s = isolated natural period due to shear distortions only;
T_f = isolated natural period due to flexural distortions only;
T_{sf} = natural period due to shear and flexure, rigid ground;
T_{sftr} = natural period due to shear, flexure, and elastically yielding ground;
T_R = natural torsional period of beam or building; rigid ground;
T_{Rg} = natural torsional period of beam or building; yielding ground;
$T_a\, T_b \cdots$ = isolated component periods; general case; and,
T_r = resultant period of several isolated component periods.

APPENDIX II

NATURAL FREQUENCIES OF A CANTILEVER BEAM SUBJECTED TO SHEAR AS WELL AS TO FLEXURE AND WITH ELASTIC CONSTRAINTS AT THE BUILT-IN END

It will be assumed that the principle of superposition of distortions is valid so that the simple theories of beam deflections may be expressed as follows:

Intensity of loading,

$$f\,(z) \qquad \text{(29}a\text{)}$$

Gradient of shear,

$$\frac{\partial V}{\partial z} = -f\,(z) \qquad \text{(29}b\text{)}$$

Variation of gradient of moment,

$$\frac{\partial^2 M}{\partial z^2} = -f\,(z) \qquad \text{(29}c\text{)}$$

Curvature of beam,[13]

$$\frac{\partial^2 y}{\partial z^2} = -\frac{M}{E\,I_x} - \frac{Y_s}{G\,A} f\,(z) \qquad \text{(29}d\text{)}$$

Variation of curvature of beam,

$$\frac{\partial^3 y}{\partial z^3} = -\frac{V}{E\,I_x} - \frac{Y_s}{G\,A}\frac{\partial f\,(z)}{\partial z} \qquad \text{(29}e\text{)}$$

Second variation of curvature,

$$\frac{\partial^4 y}{\partial z^4} = \frac{f\,(z)}{E\,I_x} - \frac{Y_s}{G\,A}\frac{\partial^2 f\,(z)}{\partial z^2} \qquad \text{(29}f\text{)}$$

The investigation will be limited to a consideration of the case in which the shearing rigidity, $\frac{G\,A}{Y_s}$, the flexural rigidity, $E\,I_x$, and the weight per unit length, w, are constants for the beam. If the beam is set into vibration, the loading intensity is dynamic, and may be expressed as follows:

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[13] "Strength of Materials," by S. Timoshenko, Pt. I, p. 187.

$$f(z) = -\frac{w}{g}\frac{\partial^2 y}{\partial t^2} \tag{30}$$

Substituting Equation (30) into Equation (29*f*), the general differential equation for the motion of the beam is obtained, as follows:

$$\frac{\partial^4 y}{\partial z^4} = -\frac{w}{g\,E\,I_x}\frac{\partial^2 y}{\partial t^2} + \frac{Y_s\,w}{g\,G\,A}\frac{\partial^4 y}{\partial z^2\,\partial t^2} = -\beta^2\frac{\partial^2 y}{\partial t^2} + \alpha^2\frac{\partial^4 y}{\partial z^2\,\partial t^2} \tag{31}$$

Equation (31) takes into account the effect of shear as well as of bending moment on the elastic distortions, but it neglects the effect of rotatory inertia on the dynamic loading. The dimensions of β are seconds per unit length^2, whereas those of α are seconds per unit length.

If one assumes that the solution of Equation (31) is given by a normal function, Z, so that:

$$y = Z\,f(t) = Z\,(E\cos p\,t + F\sin p\,t) \tag{32}$$

then Equation (32) may be substituted in Equation (31) with the result that the differential equation for Z becomes:

$$\frac{\partial^4 Z}{\partial z^4} + \alpha^2\,p^2\frac{\partial^2 Z}{\partial z^2} - \beta^2\,p^2\,Z = 0 \tag{33}$$

Equation (33) is a linear differential equation, and yields the following solution:

$$y = (A\cos q\,z + B\sin q\,z + C\cosh r\,z + D\sinh r\,z)\,f(t) \tag{34}$$

subjected to the restriction that q^2 and r^2 must satisfy the following quadratic equations,

$$q^4 - \alpha^2\,p^2\,q^2 - \beta^2\,p^2 = 0 \tag{35a}$$

and,

$$r^4 + \alpha^2\,p^2\,r^2 - \beta^2\,p^2 = 0 \tag{35b}$$

This means that:

$$q = \frac{\alpha\,p}{\sqrt{2}}\sqrt{\sqrt{1 + \frac{4\,\rho}{p^2}} + 1} \tag{36a}$$

and,

$$r = \frac{\alpha\,p}{\sqrt{2}}\sqrt{\sqrt{1 + \frac{4\,\rho}{p^2}} - 1} \tag{36b}$$

in which the units are radians per unit length. The parameter, ρ (in radians per second)^2 is defined as follows:

$$\rho = \frac{\beta^2}{\alpha^4} = \frac{g\,G^2\,A^2}{Y_s^2\,w\,E\,I_x} \tag{37}$$

Thus, for the case in which the beam distortions are due to shear only, $\rho = 0$; $r = 0$; and $q = \alpha\,p$, in radians per unit length. If the distortions of the beam were due to flexure only, the value of ρ would approach infinity; and $r = q = \sqrt[4]{\rho\,\alpha^4\,p^2} = \sqrt{\beta\,p} = u$.

If the origin of the co-ordinate axes is placed at the free end of the beam, (see Fig. 1) the following end conditions prevail: At the free end where $z = 0$, the moment must be zero, hence from Equations (29*d*) and (30):

$$\left(\frac{\partial^2 y}{\partial z^2}\right)_{z=0} = \alpha^2 \left(\frac{\partial^2 y}{\partial t^2}\right)_{z=0} \qquad (38a)$$

Moreover, at the free end the shear must be zero, hence from Equations (29*e*) and (30):

$$\left(\frac{\partial^3 y}{\partial z^3}\right)_{z=0} = \alpha^2 \left(\frac{\partial^3 y}{\partial t^2\, \partial z}\right)_{z=0} \qquad (38b)$$

At the built-in end where $z = L$ the shear, $V_{z=L}$, acting upon the ground of translatory rigidity, R_y, will produce a displacement,

$$(y)_{z=L} = \frac{V_{z=L}}{R_y} = -\frac{1}{R_y}\int_0^L f(z)\, dz = \frac{w}{g\, R_y}\int_0^L \frac{\partial^2 y}{\partial t^2}\, dz$$

$$= \tau \int_0^L \frac{\partial^2 y}{\partial t^2}\, dz \qquad (39)$$

Moreover, the slope of the beam at the built-in end is due to the shear in the beam itself as well as to the rotation of the ground. This rotation depends on the moment exerted by the beam upon the ground and on the rotatory rigidity, $R_{y\theta}$, of the ground itself. The shear contribution to the slope at $z = 0$ is:

$$\left(\frac{\partial y_s}{\partial z}\right)_{z=L} = \frac{Y_s\, V_{z=L}}{G\, A} = -\frac{Y_s}{G\, A}\int_0^L f(z)\, dz$$

$$= \frac{Y_s\, w}{g\, G\, A}\int_0^L \frac{\partial^2 y}{\partial t^2}\, dz = \alpha^2 \int_0^L \frac{\partial^2 y}{\partial t^2}\, dz \qquad (40a)$$

Whereas the contribution due to the rotation of the ground is:

$$\left(\frac{\partial y_r}{\partial z}\right)_{z=L} = \frac{M_{z=L}}{R_{y\theta}} = -\frac{1}{R_{y\theta}}\int_0^L \int f(z)\, dz\, dz$$

$$= +\frac{w}{g\, R_{y\theta}}\int_0^L \int \frac{\partial^2 y}{\partial t^2}\, dz\, dz = +\zeta \int_0^L \int \frac{\partial^2 y}{\partial t^2}\, dz\, dz \qquad (40b)$$

Consequently:

$$\left(\frac{\partial y}{\partial z}\right)_{z=L} = \left(\frac{\partial y_s}{\partial z}\right)_{z=L} + \left(\frac{\partial y_r}{\partial z}\right)_{z=L} = \alpha^2 \int_0^L \frac{\partial^2 y}{\partial t^2}\, dz$$

$$+ \zeta \int_0^L \int \frac{\partial^2 y}{\partial t^2}\, dz\, dz \qquad (41)$$

From the first two end conditions given by Equations (38),

$$C = \frac{r^2}{q^2} A \qquad (42a)$$

and,

$$D = \frac{r}{q} B \text{.........................} \quad (42b)$$

If Equations (42) are introduced into Equation (34), the following expression for y will be obtained:

$$y = \left[A \left(\cos q\, z + \frac{r^2}{q^2} \cosh r\, z \right) + B \left(\sin q\, z + \frac{r}{q} \sinh r\, z \right) \right] f\,(t) \text{..} \quad (43)$$

The constants, A and B, depending on end conditions, are complicated and will not be evaluated here, but the frequency equation, resulting from substituting Equation (43) into the expressions for the end conditions at the built-in end (that is, into Equations (39) and (41)), assumes the form:

$$\begin{aligned} & 2\,\alpha^4 \rho + 2\,\tau\,\zeta\,p^2 \\ & + (2\,\alpha^4 \rho - 2\,\tau\,\zeta\,p^2 + \alpha^4\,p^2) \cos q\,L \cosh r\,L \\ & + \left(+ \frac{\zeta}{r} - \tau\,r \right) \alpha^2\,p^2 \sqrt{1 + \frac{4\,\rho}{p^2}} \cos q\,L \sinh r\,L \\ & + \left(- \frac{\zeta}{q} - \tau\,q \right) \alpha^2\,p^2 \sqrt{1 + \frac{4\,\rho}{p^2}} \sin q\,L \cosh r\,L \\ & + \left(- \frac{\tau\,\zeta}{\sqrt{\rho}} p^2 - \alpha^4 \sqrt{\rho} \right) p \sin q\,L \sinh r\,L = 0 \text{.........} \quad (44) \end{aligned}$$

The frequency equation, of course, is rather involved, containing either explicitly or implicitly the four constants: α and β for the beam itself, and τ and ζ for the ground.

Discussion of Frequency Equation

It will now be shown that a number of particular cases for some of which the frequency equations are well known, can be derived from the general frequency formula, Equation (44).

Case a.—If the ground is assumed to be rigid in translation as well as in rotation, τ and ζ are zero and the general equation reduces to:

$$\cos q\,L \cosh r\,L - \frac{p \sqrt{\rho}}{p^2 + 2\,\rho} \sin q\,L \sinh r\,L = - \frac{2\,\rho}{p^2 + 2\,\rho} \text{....} \quad (45)$$

Therefore, Equation (45) will give the frequencies for the transverse modes of vibration of a cantilever beam rigidly built-in when shear distortions as well as flexural distortions are present (see Table 3(a)).

Case a_1.—If the problem is simplified further and the shear deformations of the beam are assumed to be negligible in comparison with the flexural deformations, Equations (37) and (36) show that $\rho = \infty$, $q = r$, and $r = \sqrt{\beta\,p} = u$. Equation (45) then reduces to the well known frequency equation for a slender cantilever beam, namely:

$$\cos u\,L \cosh u\,L = -1 \text{..................} \quad (46a)$$

TABLE 3.—RATIOS OF TRUE PERIODS, T, TO PERIODS COMPUTED ON ASSUMPTIONS OF SHEAR ONLY (T_s) AND ON ASSUMPTIONS OF FLEXURE ONLY (T_f); CANTILEVER BEAM RIGIDLY BUILT-IN

EQUATIONS TO EXPRESS MODES OF VIBRATION:

(a) Fundamental Mode of Vibration,

$$T_1 = \left(\frac{T}{T_s}\right)_1 0.288\sqrt{y_s};\quad \text{or,}\quad T_1 = \left(\frac{T}{T_f}\right)_1 0.258\sqrt{y_f}$$

(b) Second Mode of Vibration,

$$T_2 = \left(\frac{T}{T_s}\right)_2 0.0960\sqrt{y_s};\quad \text{or,}\quad T_2 = \left(\frac{T}{T_f}\right)_2 0.0412\sqrt{y_f}$$

(c) Third Mode of Vibration,

$$T_3 = \left(\frac{T}{T_s}\right)_3 0.0576\sqrt{y_s};\quad \text{or,}\quad T_3 = \left(\frac{T}{T_f}\right)_3 0.0147\sqrt{y_f}$$

(d) Fourth Mode of Vibration,

$$T_4 = \left(\frac{T}{T_s}\right)_4 0.0411\sqrt{y_s};\quad \text{or,}\quad T_4 = \left(\frac{T}{T_f}\right)_4 0.0075\sqrt{y_f}$$

Variable (see sub-caption)	VALUES OF PERIOD RATIOS FOR THE FOLLOWING MODES OF VIBRATION										
	Fundamental (Equations (a))		Second Mode (Equations (b))			Third Mode (Equations (c))			Fourth Mode (Equations (d))		
	$\left(\frac{T}{T_s}\right)_1$	$\left(\frac{T}{T_f}\right)_1$	$\left(\frac{T}{T_s}\right)_2$	$\left(\frac{T}{T_f}\right)_2$	$\frac{T_1}{T_2}$	$\left(\frac{T}{T_s}\right)_3$	$\left(\frac{T}{T_f}\right)_3$	$\frac{T_1}{T_3}$	$\left(\frac{T}{T_s}\right)_4$	$\left(\frac{T}{T_f}\right)_4$	$\frac{T_1}{T_4}$
(a) PERIOD RATIOS FOR DIFFERENT VALUES OF $\frac{y_f}{y_s}$, OR μ (IN FIRST COLUMN)											
0	1.00	....	1.00	....	....	1.00	....	....	1.00	....	....
0.25	1.09	2.43	1.06	4.94	....	1.02	8.08	....	1.01	12.00	....
0.5	1.17	1.72	1.11	3.62	....	1.04	5.78	....	1.02	7.84	....
1	1.32	1.47	1.20	2.80	....	1.07	4.23	....	1.03	5.64	....
2	1.60	1.26	1.32	2.18	....	1.11	3.09	....	1.06	4.11	....
3	1.84	1.19	1.41	1.90	....	1.15	2.61	....	1.08	3.44	....
4	2.04	1.14	1.48	1.73	....	1.19	2.32	....	1.11	3.03	....
5	2.23	1.11	1.54	1.60	....	1.22	2.16	....	1.13	2.76	....
6	2.39	1.09	1.59	1.51	....	1.25	2.00	....	1.15	2.56	....
7	2.54	1.07	1.64	1.44	....	1.28	1.90	....	1.17	2.41	....
8	2.69	1.06	1.68	1.39	....	1.31	1.82	....	1.18	2.29	....
9	2.83	1.05	1.71	1.34	....	1.34	1.75	....	1.20	2.23	....
10	2.97	1.05	1.74	1.28	....	1.36	1.71	....	1.22	2.11	....
∞	....	1.00	....	1.00	....	....	1.00	....	....	1.00	....
(b) PERIOD RATIOS FOR DIFFERENT VALUES OF THE HEIGHT-TO-DEPTH RATIO, $\frac{L}{a}$ (IN FIRST COLUMN); SOLID SQUARE CROSS-SECTION $\left(\mu = \frac{L^2}{a^2}\right)$											
1	1.32	1.47	1.20	2.80	3.30	1.07	4.23	6.17	1.03	5.64	8.97
2	2.04	1.14	1.48	1.73	4.13	1.19	2.32	8.41	1.10	3.03	13.0
3	2.83	1.05	1.71	1.34	4.96	1.34	1.75	10.6	1.20	2.23	16.5
4	3.70	1.04	1.94	1.11	5.77	1.50	1.47	12.4	1.33	1.83	19.4
(c) PERIOD RATIOS FOR DIFFERENT VALUES OF HEIGHT-TO-DEPTH RATIO, $\frac{L}{a}$ (IN FIRST COLUMN); SQUARE BOX CROSS-SECTION $\left(\mu = 0.267\frac{L^2}{a^2}\right)$											
1	1.10	2.38	1.07	4.81	3.08	1.03	7.83	5.34	1.01	10.7	7.6
2	1.34	1.43	1.21	2.72	3.33	1.08	4.09	6.20	1.04	5.49	9.02
3	1.70	1.22	1.36	2.03	3.75	1.13	2.86	7.52	1.07	3.73	11.1
4	2.09	1.13	1.50	1.68	4.19	1.20	2.28	8.72	1.10	2.91	13.3
5	2.49	1.07	1.62	1.46	4.61	1.27	1.93	9.80	1.15	2.45	15.1
6	2.92	1.05	1.73	1.30	5.06	1.35	1.71	10.8	1.21	2.14	16.9
7	3.41	1.04	1.82	1.20	5.62	1.44	1.57	11.8	1.26	1.91	18.9
∞	....	1.00	....	1.00	6.27	....	1.00	17.6	....	1.00	34.4
(d) PERIOD RATIOS FOR DIFFERENT VALUES OF THE HEIGHT-TO-DEPTH RATIO, $\frac{L}{a}$ (IN FIRST COLUMN); RECTANGULAR BOX CROSS-SECTION WITH $\frac{b}{a} = 3.1$ AND $\mu = 0.107\frac{L^2}{a^2}$											
1	1.04	4.56	1.03	7.41	3.03	1.01	12.1	5.15	1.01	16.9	7.22
2	1.14	1.95	1.10	3.96	3.11	1.04	6.20	5.48	1.02	8.52	7.83
3	1.31	1.48	1.20	2.88	3.28	1.07	4.27	6.12	1.04	5.78	8.82
4	1.53	1.31	1.29	2.31	3.56	1.10	3.29	6.96	1.06	4.44	10.1
5	1.77	1.31	1.38	1.98	3.85	1.14	2.72	7.76	1.08	3.63	11.5
6	2.02	1.16	1.47	1.76	4.12	1.18	2.35	8.56	1.09	3.07	13.0
7	2.27	1.11	1.55	1.59	4.39	1.22	2.10	9.31	1.12	2.69	14.2
8	2.52	1.08	1.63	1.46	4.64	1.27	1.90	9.92	1.16	2.44	15.2
9	2.79	1.06	1.70	1.35	4.92	1.33	1.77	10.5	1.19	2.24	16.4
10	3.08	1.05	1.75	1.25	5.28	1.39	1.67	11.1	1.23	2.07	17.5

Case a_2.—If the problem had been simplified by assuming that flexural deformations of the beam were negligible in comparison with shear deformations, Equations (37) and (36) would have given $\rho = 0$, $r = 0$, and $q = \alpha\, p$, and the frequency Equation (45) would have reduced to:

$$\cos \alpha\, p\, L = 0 \quad \text{(46b)}$$

This is the frequency equation for a stubby beam when shear alone is of importance. If the constant, α, had been made to refer to torsional vibrations instead of to translational shear vibrations, the same frequency equation would hold.

Case b.—Going to the other extreme in regard to the character of the end fastening and assuming that the cantilever beam becomes a free beam, the ground constants, τ and ζ, must approach infinity, and the general frequency equation reduces to:

$$\cos q\, L \cosh r\, L + \frac{p}{2\sqrt{\rho}} \sin q\, L \sinh r\, L = 1 \quad \text{(46c)}$$

This equation applies to the frequencies of free beams as, for instance, the bars used in xylophones. Flexure as well as shear is considered.

Case b_1.—Simplifying Equation (46*c*) further by assuming that flexure alone is of consequence:

$$\cos u\, L \cosh u\, L = 1 \quad \text{(46d)}$$

another well known result.

Case b_2.—If the free beam is very stubby, shear alone governs the frequencies and Equation (46*c*) reduces to:

$$\cos \alpha\, p\, L = 1 \quad \text{(46e)}$$

This type of frequency equation also applies to the torsional or to the longitudinal vibrations of a free bar.

Case c_1.—If the hypothetical situation of a ground, yielding elastically in translation but infinitely rigid in rotation is to be considered, $\zeta = 0$, and $\tau \neq 0$. Then for the case in which shear alone is of importance the general frequency equation (44) would reduce to:

$$\tan \alpha\, p\, L = \frac{\alpha}{p\, \tau} \quad \text{(47a)}$$

This type of equation also applies to the torsional vibrations of cantilever beams with elastic torsional end constraint (see Fig. 5).

Case c_2.—If the translatory rigidity vanishes, τ approaches infinity while the rotatory ground rigidity remains infinite (that is, $\zeta = 0$); the result for flexure alone then becomes:

$$\tan u\, L = -\tanh u\, L \quad \text{(47b)}$$

Case c_3.—The result for shear alone assumes the form:

$$\tan \alpha\, p\, L = 0 = \sin \alpha\, p\, L \quad \text{(47c)}$$

Case d_1.—An equally hypothetical case of a ground yielding elastically in rotation, but being infinitely rigid in translation would give for shear alone,

$$\tan \alpha\, p\, L = \frac{\alpha^3\, p}{\zeta} \qquad (47d)$$

Case d_2.—Again, if the rotatory rigidity were to vanish completely, ζ must approach infinity and the result is a beam with a hinge at one end. Under these circumstances flexural considerations alone would give:

$$\tan u\, L = \tanh u\, L \qquad (47e)$$

Case d_3.—Shear alone gives:

$$\tan \alpha\, p\, L = 0 \qquad (47f)$$

APPENDIX III

A CRUDE, RATIONAL ANALYSIS OF THE ELASTIC CONSTRAINING ACTION OF THE GROUND OF A CANTILEVER BEAM OR IDEALIZED BUILDING

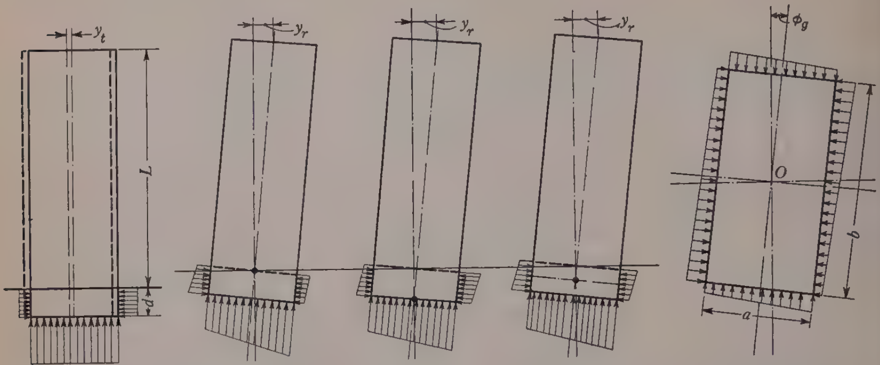
In regard to the "fixed"-end conditions of the cantilever beam, it was assumed that the medium in which it is "planted" is a semi-infinite elastic continuum; further assumptions can now be made, namely, that the inertia properties of the continuum may be neglected, and that the interaction pressures between the medium and the cantilever beam are linearly distributed over the contact surfaces involving ignorable discontinuities at edges and at corners. It is true that these assumptions are very crude, and that some doubt exists in regard to their validity even as first approximations to reality.

For purposes of analysis one may recognize three ground deformation moduli, a vertical ϵ_v, a horizontal ϵ_h, and over the base of the cantilever a shear modulus ϵ_s. These moduli differ from moduli of elasticity in being expressed in stress per total deformation; that is, in pounds per cubic inch instead of in pounds per square inch. The translatory rigidity of the ground in the Y-direction (in pounds per inch) is then given by:

$$R_y = 2\, \epsilon_h\, b\, d + \epsilon_s\, a\, b \qquad (48)$$

The corresponding horizontal translation, y_t, due to a hypothetical horizontal loading of the cantilever will be constant for the entire height of the beam (see Fig. 9(*a*)) and will be as expressed by Equation (18).

Directing attention to the rotation of the center line of the cantilever resulting from the same hypothetical loading as before, the investigator is confronted with some uncertainty in regard to the location of the center of rotation. Figs. 9(*b*), 9(*c*), and 9(*d*) indicate three possible locations of this center with the resulting pressure distributions over the contact surfaces. If the elastic restoring moment of the ground per unit angular rotation is evaluated for the three positions of the center of rotation, they will be found to increase as the center moves downward, but the resulting linear translations of the top of the cantilever, corresponding to the ground influence on a fundamental mode, will not differ as much as the angles. For this reason the question of the exact location of the rotation center may be dismissed, assuming that it lies in the plane of the ground surface as shown in Fig. 9(*b*). The rotatory rigidity of the ground for this location is given, in pound-inches per radian, by:



(*a*) ELASTIC TRANSLATION, y_t

(*b*) ELASTIC TILTING, y_r
Rotation Center at Ground Surface

(*c*) ELASTIC TILTING, y_r
Rotation Center at at End of Beam

(*d*) ELASTIC TILTING, y_r
Rotation Center Between (*b*) and (*c*)

(*e*) ELASTIC TORSION, ϕ_g

FIG. 9.—SIMPLIFIED DIAGRAMS OF GROUND PRESSURE DISTRIBUTIONS AT END OF CANTILEVER BEAM SUBJECTED TO ELASTIC DEFORMATION OF GROUND

$$R_{y\theta} = \epsilon_v \frac{a^3\, b}{12} + \epsilon_h \frac{2\, b\, d^3}{3} \dots\dots\dots\dots (49a)$$

and,

$$R_{x\theta} = \epsilon_v \frac{a\, b^3}{12} + \epsilon_h \frac{2\, a\, d^3}{3} \dots\dots\dots\dots (49b)$$

to which the corresponding horizontal translations of the top of the beam (y_r and x_r, in inches), are expressed by Equation (19).

Finally, the torsional rigidity of the ground is considered for a hypothetical twisting moment of $w\, k$ lb-in. per unit height of the beam. Fig. 9(*e*) shows the resulting pressure distributions on the vertical surfaces or basement walls of the beam, and the torsional rigidity coefficient of the ground (in pound-inches per radian) is easily shown to be:

$$R_\phi = \epsilon_h \frac{1}{6} (a^3 + b^3)\, d + \epsilon_s \frac{1}{12} (a\, b^3 + a^3\, b) \dots\dots\dots\dots (50)$$

which corresponds to an angle of twist of the beam (ϕ_g), in the ground itself which is expressed by Equation (20).

A few words in regard to the relative values of the deformation moduli are appropriate if the theory must be simplified.

If the ground medium is homogeneous and isotropic, the values of ϵ_h, ϵ_v, and ϵ_s are related to: (1) The two elastic constants, E and $\frac{1}{m}$; (2) the dimensions of the cross-section; and (3) the depth of "planting." On the other hand, if the ground medium is ordinary soil it becomes very difficult to make any theoretical predictions of the values of the ground deformation moduli.

Since the medium, or soil under the beam or building, is usually highly consolidated compared to the soil adjacent to the basement walls, it is reasonable to expect that ϵ_v is much greater than ϵ_h; the order of magnitude of the ratio, $\gamma = \frac{\epsilon_v}{\epsilon_h}$, may well lie between the values 10 and 100. The relative value of ϵ_s is perhaps equally problematic since it depends to a certain extent on the frictional adhesion of the soil to the base. However, it would be reasonable to expect that ϵ_s is of the same order of magnitude as ϵ_h. With the object of simplification in mind, and for want of better information, the investigator may choose to make ϵ_s equal to ϵ_h. The simplified expressions for the rigidities (compared with Equations (48), (49), and (50)) are then:

$$R_y = \epsilon_h \,(2\, b\, d + a\, b) \tag{51a}$$

$$R_x = \epsilon_h \,(2\, a\, d + a\, b) \tag{51b}$$

$$R_{y\theta} = \epsilon_h \frac{1}{12} b\,(\gamma\, a^3 + 8\, d^3) \tag{51c}$$

$$R_{x\theta} = \epsilon_h \frac{1}{12} a\,(\gamma\, b^3 + 8\, d^3) \tag{51d}$$

$$R_\phi = \epsilon_h \frac{1}{6}\left[(a^3 + b^3)\, d + \frac{1}{2}(a\, b^3 + a^3\, b)\right] \tag{51e}$$

The chief value of this analysis is to recognize that since there are five rigidities R_x, R_y, $R_{x\theta}$, $R_{y\theta}$, and R_ϕ expressed by the ground deformation constant, ϵ_h, and the ratio, γ, plus the dimensions of the basement, a, b, and d, the following relations are determined for the rigidities:

$$R_y = \frac{b\,(24\, d + 12\, a)}{2\,(a^3 + b^3)\, d + (a\, b^3 + a^3\, b)} R_\phi \tag{52a}$$

$$R_x = \frac{a\,(24\, d + 12\, b)}{2\,(a^3 + b^3)\, d + (a\, b^3 + a^3\, b)} R_\phi \tag{52b}$$

$$R_y = \frac{b\,(2\, d + a)}{a\,(2\, d + b)} R_x \tag{52c}$$

$$R_{y\theta} = \frac{\gamma\, a^3 + 8\, d^3}{24\, d + 12\, a} R_y \tag{52d}$$

$$R_{x\theta} = \frac{\gamma\, b^3 + 8\, d^3}{24\, d + 12\, b} R_x \tag{52e}$$

$$R_{y\phi} = \frac{b\,(\gamma\, a^3 + 8\, d^3)}{a\,(\gamma\, b^3 + 8\, d^3)} R_{x\phi} \tag{52f}$$

The hypothetical static displacements are then related as follows:

$$y_r = \frac{(12\,d + 6\,b)\,L^2}{\gamma\,a^3 + 8\,d^3}\,y_t \qquad (53a)$$

$$x_r = \frac{(12\,d + 6\,a)\,L^2}{\gamma\,b^3 + 8\,d^3}\,x_t \qquad (53b)$$

$$\phi_g = \frac{12\,R\,(2\,b\,d + a\,b)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,y_t \qquad (53c)$$

and,

$$\phi_g = \frac{12\,R\,(2\,a\,d + a\,b)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,x_t \qquad (53d)$$

If two independent rigidities can be found from a study of observed periods, the remaining three follow. It must be remembered, however, that only in rare instances are the two ground deformation constants, ϵ_h and γ, sufficient for expressing the conditions of the soil around a building, because not only geological features, but also man-made constructions, as exemplified by the proximity of other buildings, will generally call for different values of ϵ_h on the different basement walls; in addition to this complication, ϵ_v may vary over the foundation, especially if a spread footing foundation is adopted.

INFLUENCE OF THE GROUND ON THE TORSIONAL PERIODS

It was mentioned previously that flexure does not complicate the problem of determining the periods of torsion. In regard to the effect of the ground on these periods a similar simplification exists since the vertical deformation constant, ϵ_v, and, consequently, the ground rotation, do not enter into the problem. Equations (1*b*), (4*b*), and (21) enable one to define the ratio of the torsional yielding of the ground to that of the beam as follows:

$$\psi = \frac{\phi_g}{\phi_R} = \frac{12\,b\,(2\,d + a)}{2\,(a^3 + b^3)\,d + (a\,b^3 + a^3\,b)}\,\frac{Y_s\,R_t}{A}\,\frac{y_t}{y_s} \qquad (54)$$

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PAPERS

PIN-CONNECTED PLATE LINKS

BY BRUCE G. JOHNSTON,[1] ASSOC. M. AM. SOC. C. E.

SYNOPSIS

Results of 106 tests of differently proportioned steel plates are presented in this paper. The specimens were loaded in tension by means of 3-in. steel pins which passed through holes in either end of the plate. The effects of plate thickness, side and bottom edge distances, and pin clearance were studied in relation both to the general yield and the ultimate strength of the links. The "dishing" phenomenon, which greatly reduces the ultimate strength of thin plates unsupported laterally, was given particular attention. Empirical formulas are given for predicting the general yield and ultimate strength of the plates, together with formulas for producing a pin-plate connection of balanced design.

INTRODUCTION

The design of steel eye-bars has developed an accepted practice in regard to the relative proportions and distribution of material in the eye-bar head. The present investigation is concerned with pin-connected plates which do not have reduced width in the body as in the case of the forged eye-bar, but which are made simply by boring pinholes near the ends of structural steel plates. Examples of this type of connection are found in steel sheave blocks, derrick tackle, anchorage bars for cables, and various types of erection equipment. Although the plates tested differ from forged eye-bars, their action around the pinhole during the progress of failure is similar to that in the eye-bar head.

The problem of designing such pin-connected plate links was encountered on a large scale in arranging the hangers for raising the suspended span of the Quebec Bridge.[2] Tests were made both on model links and on full-sized hangers with 12-in. pins, and the results show a good correlation with those of the present tests. The general relation between plate thickness and "dishing" of thin plates was noted in the Quebec tests.

The bibliography on the subject of stresses in plates around pinholes is extensive, but most of it is concerned primarily with the study of stress dis-

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NOTE.—Written comments are requested for immediate publication; to ensure publication, the last discussion should be submitted **July 15, 1938.**

[1] Instructor, Civ. Eng., Columbia Univ., New York, N. Y.

[2] "The Quebec Bridge," Vol. 1, pp. 222 to 227, Rept. of Canadian Govt. Board of Engrs., 1919.

tribution in the elastic range. References to various photo-elastic, mathematical, and experimental studies of the stresses in eye-bar heads may be found in the work of Professor S. Timoshenko.[3] Mathematical stress analyses require an assumed distribution of bearing pressure between pin and pinhole as a starting point. The actual bearing stress distribution depends on pin clearance and varies somewhat during the application of load. The maximum stresses are localized and, for ductile materials, the initial yielding is restrained from rapid progress by surrounding low-stressed areas and, in the early stages, has no appreciable effect on the elastic behavior of the link as a whole. Furthermore, these localized stresses have practically no effect on the ultimate strength of the link under static load. Hence, the use of a ductile material such as structural steel is very advantageous.

Local yielding in the plate at the contact point, except in the case of tight-fitting pins, will occur even at working loads. The general, or over-all yield at the end of the plate, occurs at a much later stage than initial local yielding. The term, "general yield point," will be used herein to designate the load or average bearing stress at which the slope of the curve of load plotted against deformation between pin and plate (about 3 pin diameters away) is three times the initial slope.

Because of the early local yielding and associated stress redistribution, mathematical formulas for maximum local stresses based on the assumption of elastic behavior have doubtful utility in the actual design of pin-connected steel plates under steady loads. For non-ductile materials, or for ductile materials designed for repeated stress, the elastic stress distribution is of importance. An approximate mathematical solution by J. Beke[4] has been applied to experimental tests of large pin-connected members in the Hugo-Preuss Bridge, in Berlin, Germany.[5]

Beke's formula is based on the simplest possible assumptions and is applicable in the elastic range. More exact solutions become increasingly complex and, in any case, give little indication as to the actual load carried at the general yield point or ultimate. To have practical value, a mathematical solution should be based on the theory of plasticity, and should include the study of the phenomenon of plastic "dishing" of thin plates. The solution of this problem has not been attempted in the present investigation. Of first importance to the designer are the answers to the questions: "How much load will a structural unit carry without excessive deformation?" and "What is its strength at failure?" In the present instance, the answer is given in the form of empirical equations, validated by a wide range of tests, and concerned with average stresses at general yield and ultimate. The large number of relatively thin plates tested give information on the greatly reduced margin between general yield and ultimate which is caused by the "dishing" type of failure in laterally unrestrained plates.

TEST PROGRAM AND PROCEDURE

The program of 106 tests was designed to cover the critical range of variables. The same size of pin, 3 in. in diameter, was used in all tests. The four variables ranged as follows: (1) Plate thicknesses varying from $\frac{1}{8}$ to $\frac{3}{4}$ in.; (2) plate widths of 6, 8, and 10 in.; (3) edge distance behind pinhole varying from 1 in. to 3.2 in.; and (4) pin clearance varying from a tight fit to 0.2 in.

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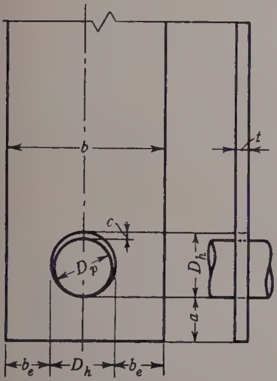
[3] "Theory of Elasticity," p. 120.

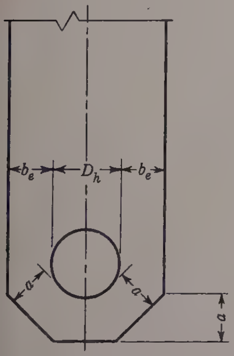
[4] "Der Eisenbau," 1921, p. 233.

[5] *Die Bautechnik*, February 12, 1932, pp. 79–83.

The dimensions of the test specimens were measured to the nearest thousandth of an inch by micrometer calipers, and are indicated in Table 1. This table shows in detail the range of variables covered. Fig. 1 shows the notation adopted in regard to pin dimensions and dimension ratios.



(a) TESTS 1 TO 103



(b) TESTS 104 TO 106

FIG. 1.—DIMENSION NOTATION FOR TEST SPECIMENS (SEE TABLE 1)

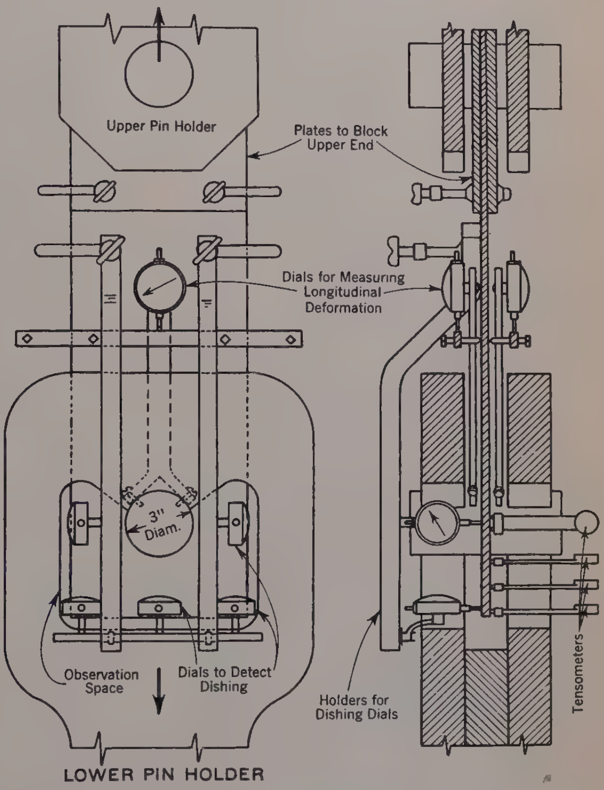

Upper Pin Holder
Plates to Block Upper End
Dials for Measuring Longitudinal Deformation
3" Diam.
Observation Space
Dials to Detect Dishing
Holders for Dishing Dials
Tensometers
LOWER PIN HOLDER

FIG. 2.—APPARATUS FOR TESTING PIN-CONNECTED STEEL PLATE LINKS

The material specified was Universal Mill Plate with rolled edges, a medium grade structural steel free from rust. All the plates of similar width and thickness were cut from the same stock with an additional section cut for tensile test coupons. The ends of the specimens were flame-cut to a length an inch or more longer than required, and were cut at the testing laboratory by hacksaw to the exact length. Two tensile coupons cut parallel with the axis of rolling and two cut transversely were made from each plate series, the results being given in Table 2.

The arrangement of the apparatus used during a typical test is shown diagrammatically in Fig. 2, and by photograph in Fig. 3. The upper pin-holder

TABLE 1.—DIMENSIONS AND TEST DATA; PIN-CONNECTED PLATE LINKS

Test No.	MEASURED DIMENSIONS, IN INCHES				SURFACE SLIP LINES; LOAD, IN KIPS		LOAD, IN KIPS†			
	Diameter of hole, D_h (1)	b_e (2)	t (3)	a (4)	First observed line* (5)	General spread of lines (6)	Local yield* (7)	General yield (8)	Initial "dishing" (9)	Ultimate strength* (10)
1	3.015	2.510	0.128	0.997	6.5 *BS*	8.0	5.6 *B*	8.5	8.0	14.5 *D*
2	3.019	2.510	0.128	1.671	7.0 *BC*	10.0	9.0 *B*	11.6	12.5+	18.0 *D*
3	3.017	2.511	0.128	2.193	10.0 *BC*	14.5	10.8 *B*	14.1	14.0	17.3 *D*
4	3.016	2.510	0.128	2.592	7.0 *BC*	15.0	11.2 *E*	15.2	13.0	17.4 *D*
5	3.015	2.512	0.128	2.881	9.0 *BC*	....	11.5 *E*	15.1	14.0	16.3 *D*
6	3.015	2.515	0.128	3.204	7.0 *BC*	....	12.0 *E*	14.4	14.0	15.0 *D*
7	3.069	2.486	0.127	0.970	6.0 *BS*	....	5.2 *B*	6.3	7.5+	15.2 *D*
8	3.070	2.482	0.128	1.700	7.5 *BC*	9.5	7.2 *B*	9.1	9.8+	15.2 *D*
9	3.069	2.482	0.128	2.185	6.0 *BC*	12.5	6.0 *B*	12.6	10.0	15.0 *D*
10	3.066	2.484	0.128	2.582	5.0 *BC*	12.5	10.5 *E*	12.4	11.0	13.5 *D*
11	3.067	2.479	0.128	2.905	5.0 *BC*	....	10.3 *B*	11.1	11.0	12.0 *D*
12	3.068	2.484	0.127	3.190	4.0 *BC*	....	11.9 *B*	12.6	11.0	13.5 *D*
13	3.017	2.517	0.190	1.018	....	11.5	10.0 *B*	10.8	10.5	25.1 *D*
14	3.014	2.510	0.189	1.700	12.0 *BS*	15.0	13.7 *B*	16.6	14.5	27.7 *D*
15	3.019	2.509	0.190	2.209	12.0 *BS*	18.0	16.4 *B*	17.7	19.0+	26.5 *D*
16	3.018	2.518	0.190	2.584	14.0 *BC*	22.0	20.3 *B*	21.3	22.0	28.9 *D*
17	3.019	2.511	0.190	2.888	16.0 *BC*	22.0	....	19.8	21.0	26.3 *D*
18	3.020	2.513	0.190	3.180	12.0 *BS*	17.0+	19.0 *E*	21.5	20.0	26.0 *D*
19	3.073	2.484	0.190	1.008	4.0 *BS*	7.5	4.2 *B*	9.7	7.0	25.3 *DF*
20	3.074	2.478	0.189	1.889	9.0 *BC*	14.5	12.1 *B*	14.1	14.5	29.7 *D*
21	3.074	2.479	0.189	2.206	5.0 *BC*	16.0	15.8 *B*	18.5	18.5+	27.0 *D*
22	3.074	2.479	0.188	2.606	7.0 *BC*	18.0	20.0 *E*	21.5	....	26.4 *D*
23	3.075	2.473	0.189	2.880	....	20.0	16.0 *E*	18.2	16.0	22.0 *D*
24	3.074	2.480	0.188	3.208	10.0 *BC*	18.0+	....	20.0	20.0	24.4 *D*
25	3.017	2.485	0.254	1.016	13.0 *BS*	20.0	12.3 *B*	16.8	....	38.0 *FB*
26	3.018	2.483	0.254	1.697	21.0 *BS*	23.0	19.1 *B*	23.2	20.0	42.5 *D*
27	3.016	2.486	0.255	2.216	15.0 *BC*	....	24.4 *B*	28.2	27.0	43.4 *D*
28	3.019	2.480	0.254	2.619	13.0 *BC*	31.0	26.2 *E*	29.7	29.0	43.4 *D*
29	3.018	2.483	0.255	2.915	15.0 *BC*	27.0	38.3 *B*	31.0	25.0	43.3 *D*
30	3.019	2.481	0.253	3.219	11.0 *BC*	31.0	29.4 *E*	32.0	34.0	41.0 *D*
31	3.072	2.458	0.256	0.998	7.0 *BC*	12.0+	8.1 *B*	12.4	10.0	36.9 *FB*
32	3.069	2.460	0.258	1.685	10.0 *BC*	20.0	15.2 *B*	19.5	20.0	49.0 *FB*
33	3.069	2.460	0.257	2.211	15.0 *BS*	26.0	21.1 *B*	25.8	25.0	48.2 *D*
34	3.072	2.452	0.256	2.590	9.0 *BC*	24.0+	24.2 *B*	27.8	27.0	43.0 *D*
35	3.073	2.459	0.257	2.908	9.0 *BC*	30.0	28.3 *B*	28.0	31.0	41.5 *D*
36	3.071	2.459	0.257	3.175	9.0 *BC*	27.0	25.8 *B*	29.0	29.0	39.6 *D*
37	3.016	2.493	0.373	0.997	17.0 *BS*	23.0	15.0 *B*	21.6	None	53.7 *FB*
38	3.016	2.495	0.372	1.709	20.0 *BC*	30.0	24.3 *B*	29.4	None	72.0 *FB*
39	3.018	2.489	0.373	2.201	21.0 *BC*	36.0	30.4 *B*	36.5	38.0+	76.5 *D*
40	3.017	2.491	0.374	2.593	21.0 *BC*	40.0	34.5 *E*	40.0	46.0+	76.5 *D*
41	3.019	2.489	0.374	2.896	18.0 *BC*	49.0	37.8 *B*	42.0	48.0+	73.4 *D*
42	3.017	2.492	0.371	3.199	21.0 *BC*	....	39.6 *E*	43.0	42.0	75.7 *D*
43	3.066	2.466	0.373	0.959	13.0 *BC*	16.0	11.6 *B*	17.0	None	58.9 *FB*
44	3.066	2.474	0.371	1.681	11.6 *BC*	26.0	21.7 *B*	24.5	None	78.0 *FB*
45	3.066	2.469	0.371	2.193	25.0 *BC*	32.0	27.4 *B*	32.5	34.0+	78.5 *D*
46	3.069	2.467	0.371	2.594	30.0 *BC*	36.0	31.2 *B*	36.5	39.0+	78.5 *D*
47	3.067	2.465	0.372	2.910	16.0 *BC*	38.0	29.0 *B*	39.5	42.0+	73.00
48	3.066	2.469	0.371	3.200	16.0 *BC*	40.0	35.5 *B*	40.5	40.0+	70.20
49	3.014	2.506	0.496	1.019	....	30.0	22.1 *B*	28.5	None	73.4 *F*
50	3.014	2.514	0.500	1.700	18.0 *BC*	39.0	36.0 *B*	40.0	None	101.8 *F*
51	3.016	2.510	0.495	2.210	35.0 *BS*	42.0	36.0 *B*	47.0	....	....
52	3.014	2.511	0.498	2.580	27.0 *BC*	....	46.4 *B*	54.5	Ult.	133.4 *D*

* *B* = below pin; *C* = crushing; *D* = "dishing"; *E* = side of pin; *F* = fracture; and *S* = shear.
† 1 kip = 1 000 lb.

passed through the upper fixed head of the testing machine where it engaged transverse bars by means of lugs which allowed freedom of motion of the holder. The lower holder passed through the moving head of the testing machine and was open at the side to allow observation of the test. The upper end of the test specimen was blocked by plates on either side to ensure failure at the lower pinhole. The holders were built entirely by flame-cutting and welding steel slabs.

TABLE 1.—(*Continued*)

Test No.	Measured Dimensions, in Inches				Surface Slip Lines; Load, in Kips		Load, in Kips†			
	Diameter of hole, D_h (1)	b_e (2)	t (3)	a (4)	First observed line* (5)	General spread of lines (6)	Local yield* (7)	General yield (8)	Initial "dishing" (9)	Ultimate strength* (10)
53	3.014	2.496	0.492	2.900	15.0 *BC*	46.0	46.0 *E*	53.5	Ult.	128.3 *D*
54	3.014	2.509	0.499	3.215	30.0 *BC*	52.0	49.8 *E*	58.0	“	126.4 *D*
55	3.067	2.479	0.494	0.993	20.0 *BC*	22.0	14.5 *B*	21.0	“	72.6 *D*
56	3.067	2.471	0.494	1.685	22.0 *BC*	31.0	28.5 *B*	33.6	None	101.5 *FB*
57	3.066	2.480	0.494	2.174	25.0 *BC*	44.0	35.3 *B*	45.0	“	118.9 *FB*
58	3.066	2.475	0.495	2.601	20.0 *BC*	48.0	35.0 *B*	47.5	“	138.2 *FB*
59	3.066	2.478	0.497	2.902	25.0 *BC*	51.0	39.5 *B*	50.8	Ult.	130.4 *D*
60	3.069	2.475	0.498	3.183	25.0 *BC*	52.0	42.0 *E*	57.0	“	121.2 *D*
61	3.015	2.494	0.745	0.988	30.0 *BC*	....	....	38.0	None	108.4 *FB*
62	3.016	2.505	0.747	1.702	30.0 *BC*	....	....	58.0	“	150.6 *FB*
63	3.016	2.491	0.746	2.176	24.0 *BC*	....	....	66.8	“	162.0 *FB*
64	3.016	2.492	0.745	2.579	....	....	....	72.6	“	185.9 *FB*
65	3.019	2.497	0.747	2.906	....	....	....	80.5	“	204.0+
66	3.017	2.490	0.745	3.207	42.0 *BC*	....	....	82.0	“	204.0+
67	3.067	2.465	0.745	1.010	....	....	....	31.2	“	111.6 *FB*
68	3.067	2.459	0.742	1.679	....	....	....	43.5	“	147.2 *FB*
69	3.067	2.465	0.745	2.215	25.0 *BC*	....	....	54.6	“	172.8 *FB*
70	3.067	2.473	0.748	2.607	....	....	....	64.0	“	200.0+
71	3.065	2.464	0.745	2.916	30.0 *BC*	....	....	73.0	“	200.0+
72	3.070	2.475	0.746	3.186	42.0 *BC*	....	....	76.0	“	200.0+
73	3.024	1.471	0.188	0.981	11.0 *BS*	15.0	7.4 *B*	11.8	12.5+	24.4 *D*
74	3.019	1.476	0.187	1.687	10.0 *BS*	15.0+	12.8 *E*	16.6	17.0+	28.4 *D*
75	3.018	1.473	0.187	2.213	14.0 *ES*	20.0	18.0 *B*	20.0	20.0+	26.2 *D*
76	3.019	1.475	0.188	2.611	12.0 *BC*	22.0	11.2 *E*	20.8	19.7	24.0 *D*
77	3.018	1.475	0.188	2.909	15.0 *ES*	22.0	13.2 *E*	21.0	18.0	25.4 *D*
78	3.019	1.474	0.187	3.205	12.0 *BC*	22.0	17.3 *E*	22.0	21.0	24.8 *D*
79	3.018	1.510	0.514	0.978	21.0 *BS*	36.0	24.3 *B*	33.0	None	64.6 *FB*
80	3.016	1.509	0.512	1.674	24.0 *BC*	....	39.8 *B*	44.0	“	98.6 *FE*
81	3.017	1.504	0.515	2.201	....	49.0	43.0 *E*	55.0	“	100.5 *FE*
82	3.016	1.506	0.512	2.571	35.0 *BC*	53.0	38.0 *E*	58.0	“	99.8 *FE*
83	3.015	1.508	0.513	2.905	35.0 *BC*	....	46.0 *E*	58.0	“	100.9 *FE*
84	3.015	1.506	0.513	3.210	25.0 *BC*	....	45.0 *E*	58.8	“	100.5 *FE*
85	3.018	3.487	0.192	0.988	8.0 *BS*	....	10.0 *B*	11.3	11.0+	25.5 *D*
86	3.018	3.487	0.190	1.711	12.0 *BC*	14.0	13.9 *B*	15.2	15.0+	29.2 *D*
87	3.020	3.480	0.190	2.216	12.0 *BC*	....	16.2 *B*	19.6	20.0+	28.1 *D*
88	3.020	3.480	0.190	2.587	15.0 *BS*	21.0	17.0 *B*	20.2	22.0+	28.1 *D*
89	3.024	3.488	0.191	2.906	12.0 *BC*	21.0	19.4 *B*	21.8	18.0	27.6 *D*
90	3.023	3.475	0.190	3.198	11.0 *BC*	21.0	18.6 *B*	21.5	20.0	27.0 *D*
91	3.016	3.485	0.508	1.008	20.0 *BC*	....	21.4 *B*	28.2	None	83.8 *FB*
92	3.015	3.487	0.507	1.731	28.0 *BC*	....	35.6 *B*	41.0	“	113.3 *FB*
93	3.015	3.485	0.509	2.191	20.0 *BC*	....	43.5 *B*	50.0	“	129.2 *FB*
94	3.020	3.481	0.507	2.573	20.0 *BC*	....	49.0 *B*	56.0	“	139.8 *FB*
95	3.017	3.482	0.507	2.897	....	....	46.5 *B*	59.0	Ult.	147.5 *D*
96	3.025	3.481	0.507	3.190	....	....	50.0 *B*	62.0	“	149.0 *D*
97	3.001	2.507	0.500	2.888	44.0 *BS*	64.0	....	62.4	“	130.0 *D*
98	3.007	2.502	0.502	2.883	49.0 *BS*	57.0	....	59.0	“	129.9 *D*
99	3.028	2.486	0.492	2.881	33.0 *BC*	57.0	....	58.5	“	127.7 *D*
100	3.058	2.480	0.501	2.895	17.0 *BC*	55.0	....	58.0	“	131.2 *D*
101	3.100	2.447	0.495	2.870	20.0 *BC*	56.0	....	55.0	“	132.5 *D*
102	3.137	2.441	0.496	2.887	20.0 *BC*	56.0	....	55.0	“	128.7 *D*
103	3.200	2.400	0.492	2.935	16.0 *BC*	52.0	....	55.0	“	122.6 *D*
104‡	3.000	2.495	0.492	2.902	49.0 *BS*	61.0	....	58.0	“	122.9 *D*
105‡	3.058	2.480	0.502	2.908	22.0 *BC*	56.0	....	58.0	“	129.7 *D*
106‡	3.200	2.402	0.492	2.897	12.0 *BC*	54.0	....	51.5	None	151.2 *FB*

‡ Corners clipped at 45 degrees.

Four different observations were made at successive loads during each test, as follows:

(1) Longitudinal deformation between pin and plate was measured by two Ames dials $\left(\frac{1}{10\,000}\right)$ attached rigidly to arms which were fixed to the pins by set screws as shown in Fig. 2. The plungers of these dials engaged bars at-

TABLE 2.—PHYSICAL PROPERTIES OF MATERIALS BASED ON THE AVERAGE RESULTS OF TWO TESTS

Plate Nos.	LONGITUDINAL					TRANSVERSE				
	Yield point stress/1 000	Ultimate strength/1 000	Percentage reduction of area	Percentage elongation in 2 inches	Modulus of elasticity, $\frac{E}{10^6}$	Yield point stress/1 000	Ultimate strength/1 000	Percentage reduction of area	Percentage elongation in 2 inches	Modulus of elasticity, $\frac{E}{10^6}$
1–12	41.1	59.6	62.4	40.3	30.8	45.6	62.0	61.1	28.0	31.0
13–24	41.3	57.9	69.0	46.7	30.3	41.0	59.3	61.4	38.2	30.0
25–36	41.6	63.7	62.6	47.5	28.4	40.2	64.4	45.3	34.5	29.5
37–48	39.4	63.4	62.7	50.0	29.1	37.4	63.3	47.5	34.8	30.8
49–60	38.5	63.4	59.2	51.8	29.3	39.0	63.3	30.3	30.0	30.1
61–72	31.3	60.2	63.3	56.8	28.9	31.3	61.5	50.3	41.5	29.1
73–78	42.0	62.6	66.6	42.2	30.6	. . . .	63.8	57.4	32.5	30.0
79–84	39.6	62.6	62.7	49.3	29.7	38.4	63.6	33.9	20.0	29.9
85–90	41.0	59.0	71.8	50.0	29.1	39.3	58.5	57.2	37.5	30.3
91–96	36.5	63.1	60.5	51.3	29.2	36.1	64.3	27.1	38.5	28.8
97–106	41.0	65.5	60.8	49.0	30.2	39.1	65.6	35.3	27.5	29.2
Average	39.2	61.6	64.1	48.6	29.5	38.7	62.4	47.2	33.6	29.9

tached to the test pieces by pointed screws about 8.5 in. from the center of the lower pin. The graph of the longitudinal deformation, plotted as abscissa with the load as ordinate, was used as a basis for determining the general yield point of the links. The point of general yield was taken as the load at which the slope of the load-deformation curve was three times that in the elastic range.

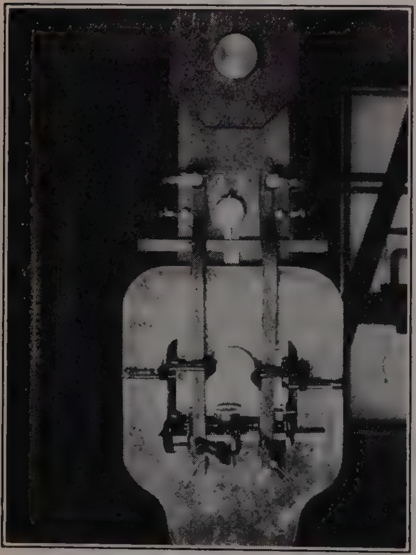


FIG. 3.—GENERAL VIEW OF TEST ARRANGEMENT, WITH DIALS PLACED TO DETECT "DISHING"

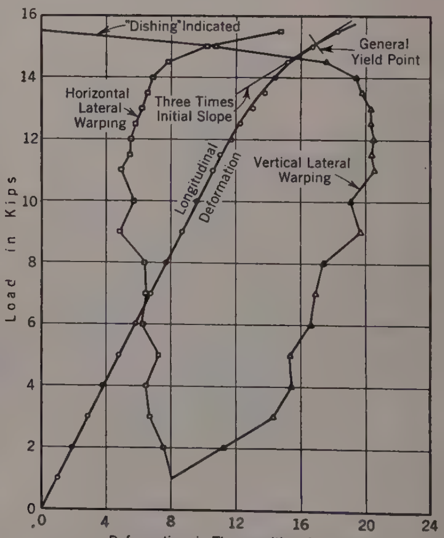


FIG. 4.—LOAD-DEFORMATION GRAPHS FOR TEST NO. 5

(2) Five Ames dials fixed to two main arms and one cross-piece engaged the lower surface of the test piece in order to measure any lateral movement

which would indicate the beginning of "dishing." The arms for holding these dials were fixed rigidly to the center of the test specimen, the arrangement being shown in Fig. 2. There were three points along two vertical lines, therefore, and three across the lower face of the plate. A study of these dial readings during successive load increments indicated any lateral movement along these three lines.

(3) Strains in the pin-plate material were measured along critical sections by means of Huggenberger tensometers, which were calibrated by an interferometer device designed and built by Raymond D. Mindlin, Jun. Am. Soc. C. E. The tensometers served two functions: (1) To obtain the stress distribution across the net section within an elastic range; and (2) to detect stress redistribution as indicated by variations in the slope of the strain curves. This, of course, does not necessarily indicate that the limit of proportionality had been exceeded at the exact point where the strain is measured. The tensometers were also used in determining the modulus of elasticity of the steel in the tensile coupon tests.

(4) The surface slip lines which are visible in the mill scale of structural steel at points of over-strain were accentuated by coating the surfaces of the plate at the lower pinhole with whitewash. The progress of the slip lines was recorded during successive load increments, and photographs were taken of all typical patterns. The whitewash consisted of 35 grams of hydrated lime mixed with 50 cu cm of water.

A similar routine was followed in all the tests, and mimeographed forms were used for recording dimensions, dial and tensometer readings, and the successive development of surface slip-lines.

Test Results

Types of Failure.—The general yield point, arbitrarily determined in every test from the load-deformation curve, was preceded by surface evidence of local failure below the pin in every instance except Tests Nos. 75 and 77 (see Table 1), these being two of the twelve tests of narrow plates 6 in. wide. Considerable surface slip-line development, extending over a relatively large area of the plate, was usually in evidence before the general yield point.

The type of ultimate failure in the pin-connected plate depends on the relative proportions of the plate at the pinhole. Three general and overlapping classifications may be made: (1) Tension failure in the net section at one side of the pin; (2) crushing and shearing failure below the pin, in some cases followed by a tearing fracture in "hoop" tension after considerable deformation; and (3) "dishing" failure of thin plates which are laterally unrestrained.

Typical Tests.—Failure by "dishing" at very low load is typified by Test No. 5, Table 1. The load-deformation curve, to determine the general yield point, and the curves of lateral and vertical warping indicative of "dishing" are shown in Fig. 4. A photograph of the surface slip-line pattern after "dishing" of Test No. 5 is shown in Fig. 5. This plate "dished" at a load of only 1 200 lb above the general yield point, and, in such cases, the slip lines appeared only on the convex surface. The "dishing" itself is not evident in this side view,

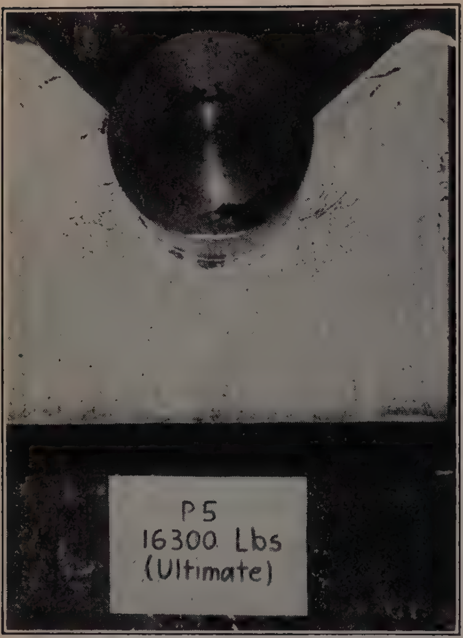


Fig. 5.—Test No. 5 After Failure by "Dishing"; Showing Slip-Lines on Convex Surface

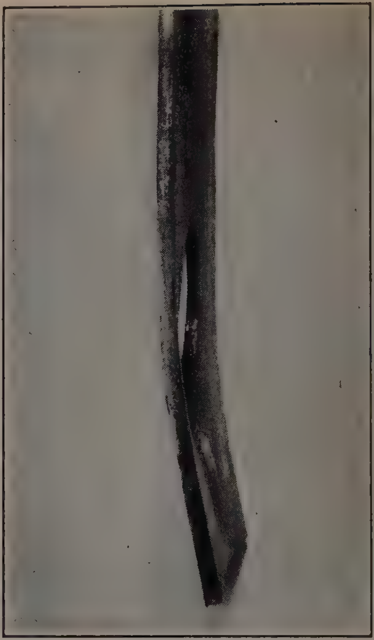


Fig. 6.—Shape of Test Piece After Failure by "Dishing"

but is clearly indicated in Fig. 6, showing the shape of the plate after complete failure by "dishing."

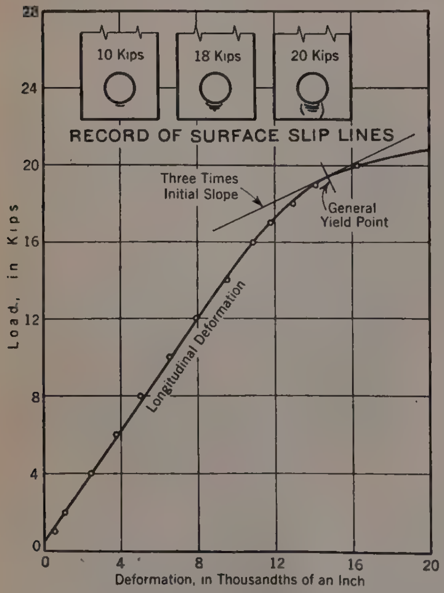


Fig. 7.—Test No. 32; Load Deformation Graph and Surface Slip-Line Record

The test of Plate No. 32 is typical of failure behind the pinhole, due to insufficient edge distance, *a*. Fig. 7 gives the load-deformation curve for this test and shows sketches similar to those taken in every test to record the progressive growth of surface slip-lines. Fig. 8 illustrates the further spread of the surface slip-lines at a load of 26 kips, and Fig. 9 shows the fracture after failure. The condition of the whitewash in Fig. 9 shows the completely plastic region below the pin prior to failure and the relative absence of surface slip-lines along the sides and above the pinhole. Fig. 10 shows an end fracture in another test of more balanced design and greater plastic elongation. Fig. 11 shows a surface slip-line pattern near the ultimate strength in

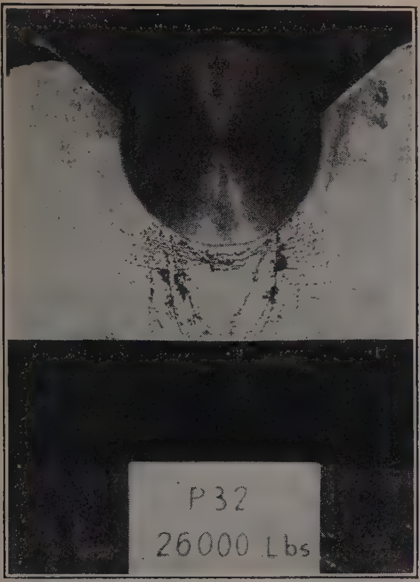


FIG. 8.—TEST NO. 32 AT 26 KIPS

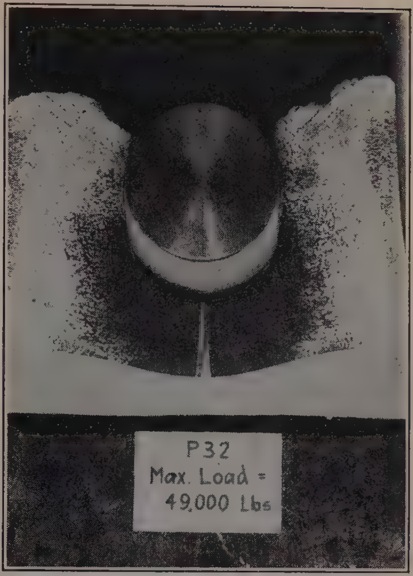


FIG. 9.—TEST NO. 32 AFTER FRACTURE BEHIND THE PIN

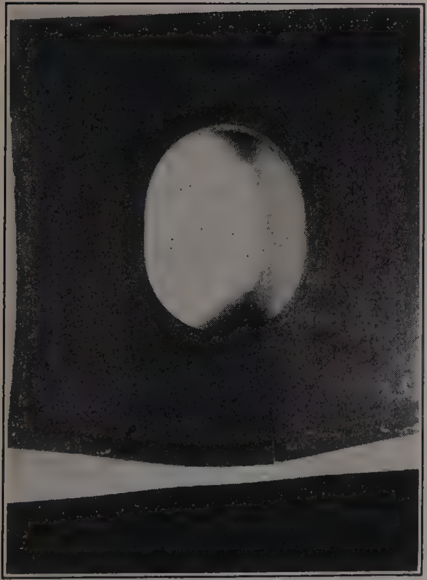


FIG. 10.—TEST NO. 64 AFTER PARTIAL FAILURE BEHIND THE PIN

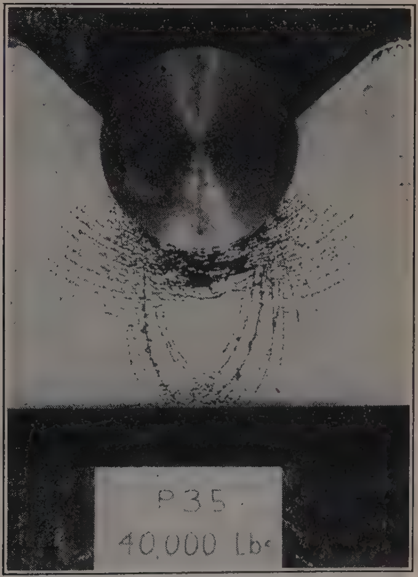


FIG. 11.—TEST NO. 35 AT 40 KIPS AND JUST PRIOR TO "DISHING"

Plate No. 35, similar to Plate No. 32, but of more balanced design. This plate was the same in every respect as Plate No. 32 with the exception of greater edge distance, *a*, with the result that considerable local crushing below the pin developed before the final spread of the slip-lines. The slip-lines indicated in Fig. 11 were duplicated on both sides of the plate. At a load of 41.5 kips, the plate "dished" causing a slip-line pattern similar to that in Fig. 5 to be superposed on the convex side. Unlike Test No. 5, however, Plate No. 35 was thick enough to develop considerable reserve strength between the general yield stress and ultimate failure by "dishing."

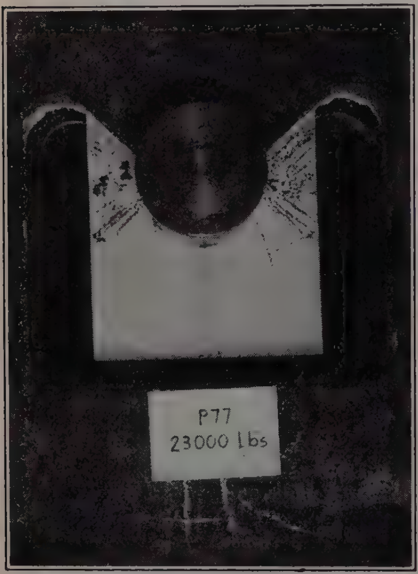


FIG. 12.—TEST NO. 77 AT 23 KIPS, SHOWING YIELDING AT THE SIDE

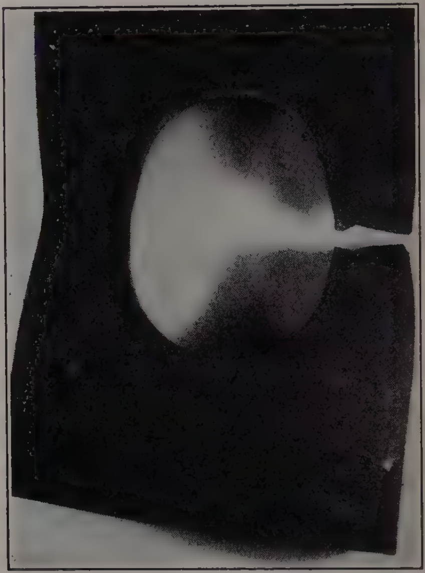


FIG. 13.—TEST NO. 82 AFTER FRACTURE AT SIDE OF PINHOLE

With one exception the series of plates which were 6 in. wide failed either in the net section in tension, or by "dishing." Fig. 12 shows the slip-line pattern soon after the general yield point of a 6-in. by $\frac{3}{16}$-in. plate prior to ultimate failure by "dishing," and Fig. 13 shows the side fracture, at ultimate strength of a 6-in. by $\frac{1}{2}$-in. plate.

Stress Distribution.—The stress distribution in the plate around the pinhole is complex, and may best be studied within the elastic range by means of photo-elastic tests. Complete studies of the stress pattern and distribution in a plate around a pinhole have been made by previous investigators.[6]

The maximum stresses are the localized contact compressive stresses at the point of bearing of pin on plate, and the concentration of tensile stress at each side of the pinhole. Except in the case of tight-fitting pins, the localized contact stresses produce the first surface evidence of local yielding. For a discussion of the subject of contact stresses reference may be made to an in-

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[6] *Journal*, Franklin Inst., Vol. 199, 1925, p. 289; and *Bulletin*, Aero. Research Inst., Tokyo Univ., September 12, 1926.

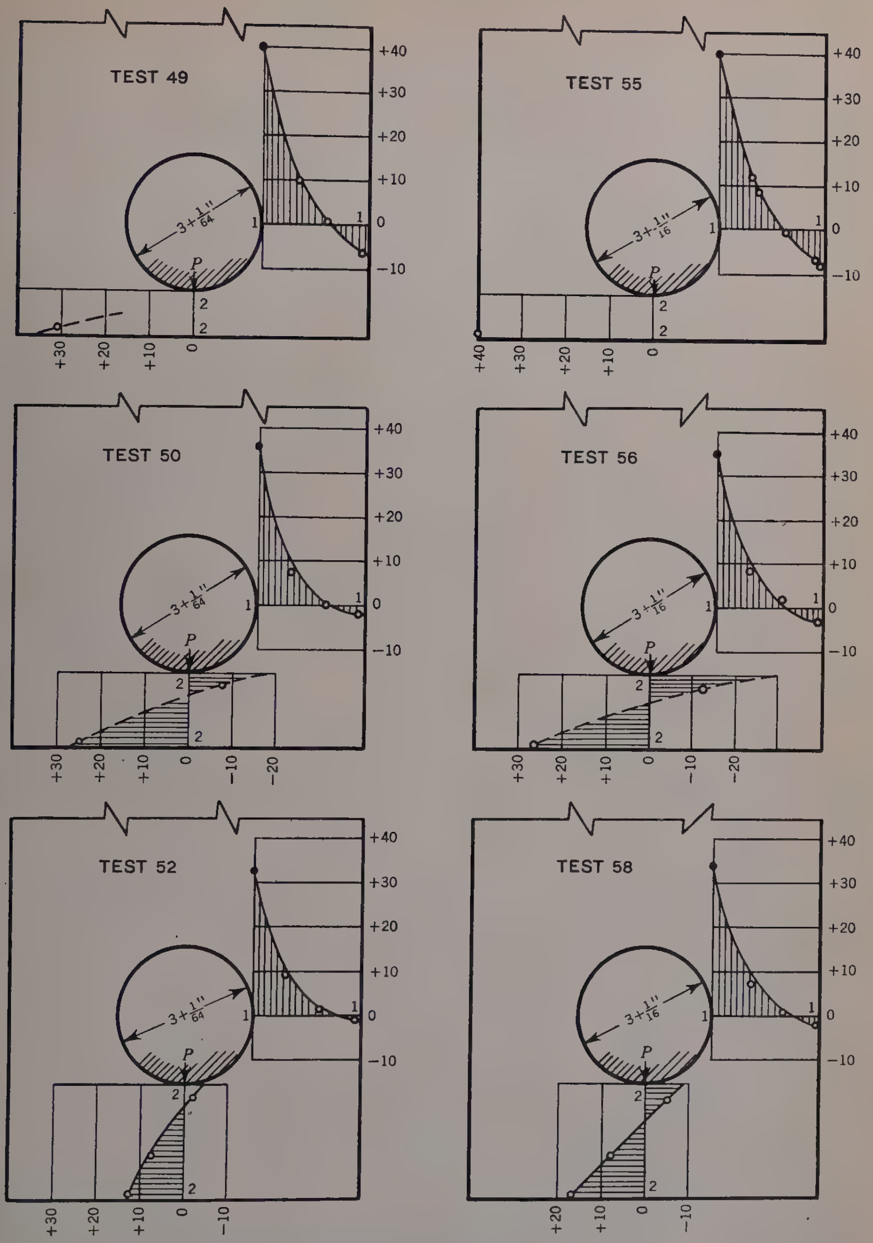

Fig. 14.—Strain Time Modulus of Elasticity Normal to the Horizontal and Vertical Lines by Tensometers ($P = 20$ Kips)

vestigation of stresses under bridge rollers reported[7] in 1936, by V. P. Jensen, Assoc. M. Am. Soc. C. E.

Insufficient clearance around the pin made it impossible to place tensometers for measuring strains at the exact locations of maximum stress. The tensometers were useful in all tests, however, in detecting initial stress redistribution which indirectly determined loads at which local yielding had occurred at points of maximum stress. Additional tensometers were used in some of the tests to determine the strain distribution along vertical and horizontal lines (see Fig. 2). The results are indicated in Fig. 14 for six tests on 8-in. by 0.5-in. plates with a varying distance, a, and two different pin clearances. The values indicated in Fig. 14 are the products of the strain in one direction multiplied by the modulus of elasticity, and are not true stresses, although very nearly so along Line 1–1, where the normal horizontal stresses are negligible. The maximum value of the vertical strain times the modulus of elasticity, E, normal to Section 1–1 adjacent to the pin was determined indirectly by extending from the measured values a stress area that would be in equilibrium with the known total load on the section. The contact stresses below the pin vary rapidly and could not have been measured satisfactorily over the half-inch gage length available in a Huggenberger tensometer, even if clearance had been available. The strains normal to the vertical section, 2–2, were measured at as many points as possible and the products of these strains and the modulus of elasticity, E, are also indicated in Fig. 14. The true stresses normal to Section 2–2 in the vicinity of the pin are considerable larger than the values of strain times the modulus of elasticity, E, because of the unmeasured contact stresses.

It is important to note that considerable local yielding, either below or to the side of the pinhole, was always evidenced before the general yield point of the plate was noted, and that stress concentrations had little effect upon the ultimate strength.

Pin Clearance.—A special series of tests was made in which clearance between pin and pinhole was the only variable. The load-deformation curves were extended far beyond the general yield point in these tests and three typical curves are shown in Fig. 15. The results of this series of tests are discussed in the general "Summary of Test Results" which follows.

Summary of Test Results.—Table 1 presents an abbreviated summary of the results of all the tests. A detailed explanation of the various columns in this table follows: Columns (1), (2), (3), and (4) give measured dimensions of the specimens before test, at the end near the observed pinhole, based on the average value of micrometer readings made to the nearest 0.001 in.

Column (5) indicates the load at which the first surface slip-line was noted, together with its location and type. The first letter, B or E, indicates whether the line was below or to one side of the pin. The second letter (S or C) indicates whether the flaking was a definite line, indicating slip or shear along a certain path, or general flaking below the pin, indicative of compression yield by crushing (see Fig. 11 for illustrations of both types). In this column, of which only a qualitative interpretation may be made, the trend of the lowest noted loads is most nearly correct, since the whitewash at times had enough cohesion to remain in place after yielding had commenced.

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[7] *Bulletin*, Iowa Eng. Soc., Vol. XI, No. 4, October, 1936, p. 9.

Column (6), Table 1, gives the load at which surface indication of failure had become general. By this is meant either the presence of several lines extending from the pinhole through to the end or side of the plate, or a general network of lines developed in the vicinity of the pinhole. The same qualifications must be made regarding interpretation as in the case of Column (5). Column (7) gives the first indication of general stress redistribution due to local yielding, either to one side (E) or below (B) the pinhole. The noted load in each case is the limit of proportionality of the tensometer readings plotted against load.

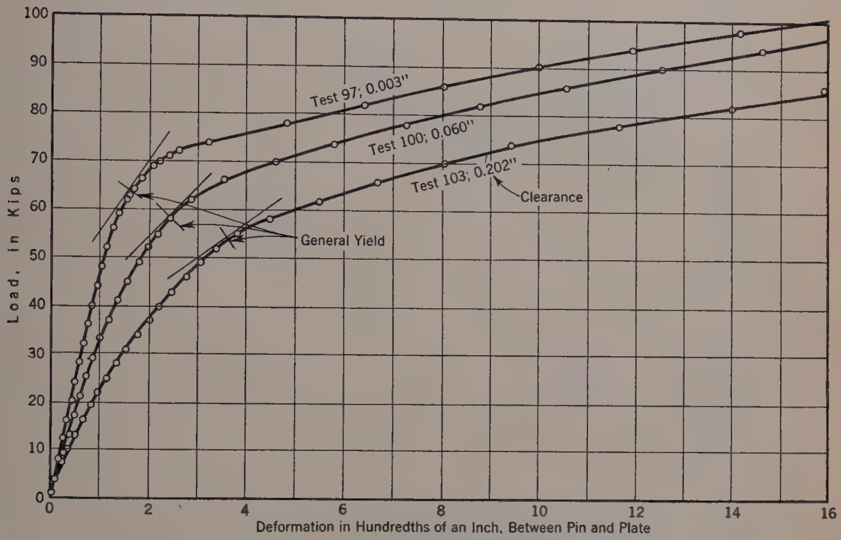
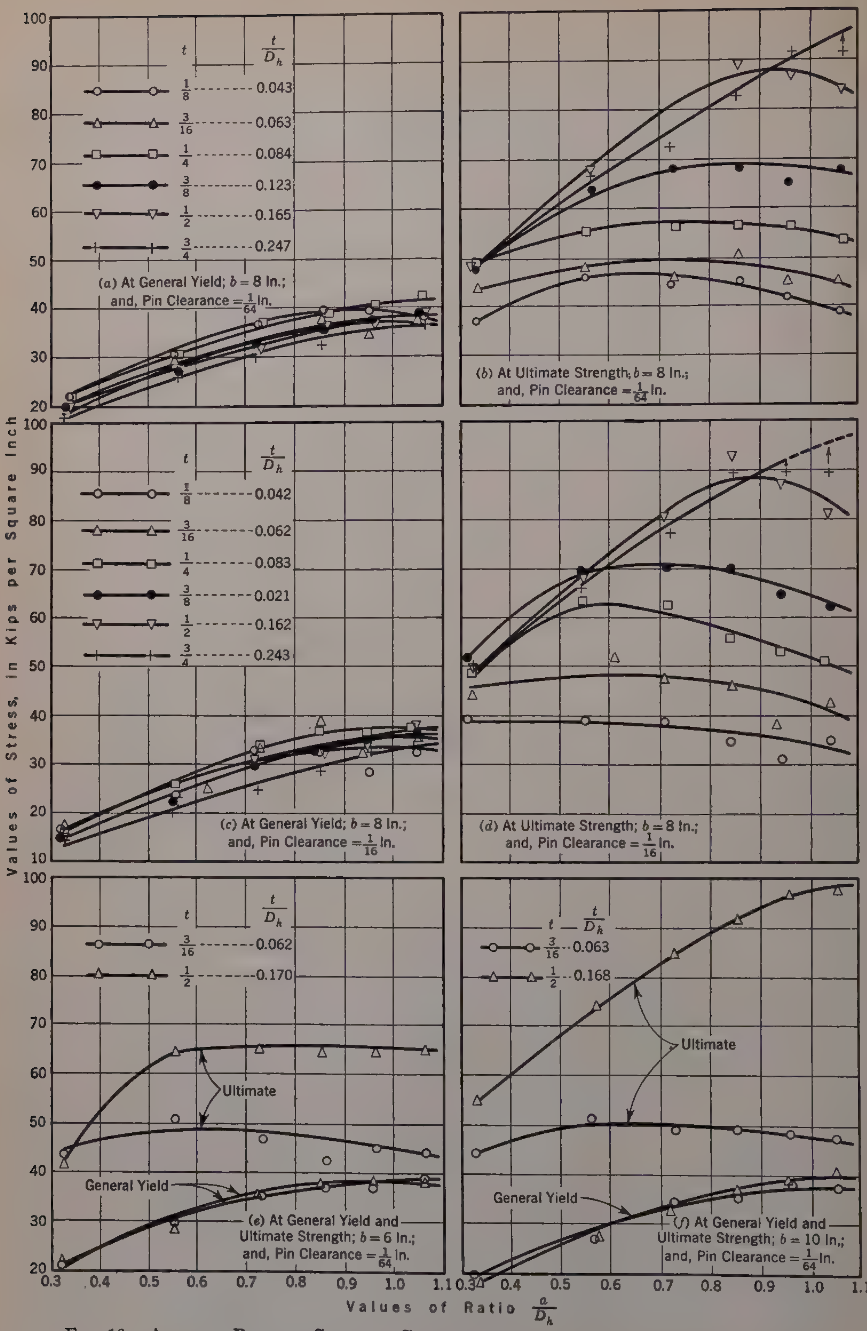


FIG. 15.—LOAD-DEFORMATION CURVES WITH PIN CLEARANCE AS THE ONLY VARIABLE

Column (8), Table 1, gives the load at the arbitrarily determined "general yield" strength at which point the longitudinal deformation between pin and plate was progressing at three times the initial rate; Column (9) indicates load at the start of "dishing," as detected by the limit of proportionality of the differences between the readings of the lateral dials, plotted against load; Column (10) gives the maximum load either at fracture or "dishing," in the latter case, just prior to large falling off of load during the progress of "dishing."

In Tests Nos. 49 to 106, the set readings were taken at five or more different load intervals to give a complete curve of set plotted against load. These data are omitted to conserve space but the permanent set in all tests followed closely the curve indicated in Fig. 19(b).

Graphical Summary of Test Results.—Average bearing stress at the general yield point, and the ultimate strength for the plates 8 in. wide (Tests Nos. 1 to 72) are plotted in Figs. 16(a) to 16(d), with the ratio, $\frac{a}{D_h}$, as abscissa. Figs.



(a) At General Yield; $b = 8$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

(b) At Ultimate Strength; $b = 8$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

(c) At General Yield; $b = 8$ In.; and, Pin Clearance $= \frac{1}{16}$ In.

(d) At Ultimate Strength; $b = 8$ In.; and, Pin Clearance $= \frac{1}{16}$ In.

(e) At General Yield and Ultimate Strength; $b = 6$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

(f) At General Yield and Ultimate Strength; $b = 10$ In.; and, Pin Clearance $= \frac{1}{64}$ In.

FIG. 16.—AVERAGE BEARING STRESSES CORRESPONDING TO VALUES OF THE RATIO, $\frac{a}{D_h}$.

16(*e*) and 16(*f*) give the same results for the 6-in. and 10-in. plates, Tests Nos. 73 to 96, inclusive.

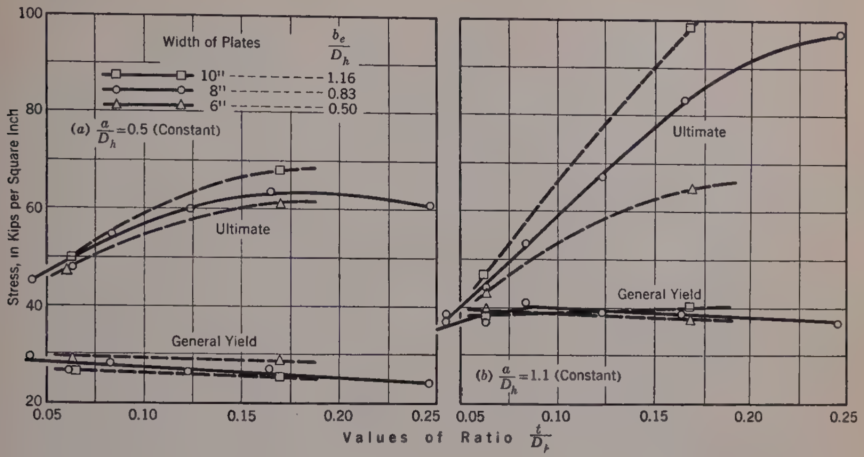


FIG. 17.—AVERAGE BEARING STRESSES AT THE GENERAL YIELD POINT AND ULTIMATE STRENGTH, WITH VARYING RATIO, $\frac{t}{D_h}$

Fig. 17 gives average bearing stresses plotted against the variable thickness ratio, $\frac{t}{D_h}$, for Tests Nos. 1 to 96, Table 1, and for two different ratios, $\frac{a}{D_h}$, of 0.5 and 1.1. The values plotted were taken from the curves in Fig. 16.

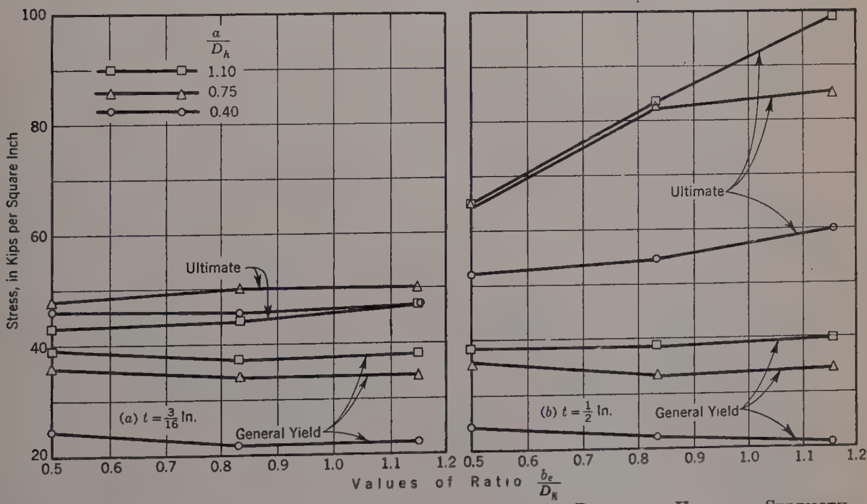


FIG. 18.—AVERAGE BEARING STRESSES AT THE GENERAL YIELD POINT AND ULTIMATE STRENGTH, WITH VARYING RATIO, $\frac{b_e}{D_h}$

In Fig. 18 the average bearing stress is plotted against the ratio, $\frac{b_e}{D_h}$, as abscissa for all the tests of $\frac{3}{16}$-in. and $\frac{1}{2}$-in. plates.

Fig. 19(*a*) gives the average bearing stress at the general yield point and ultimate strengths for Tests Nos. 97 to 106 (Table 1), in which the pin clearances are the variable, and are plotted as abscissa. Fig. 19(*b*) shows the relation between permanent set and variable pin clearance for the same series at different bearing stresses and at the general yield.

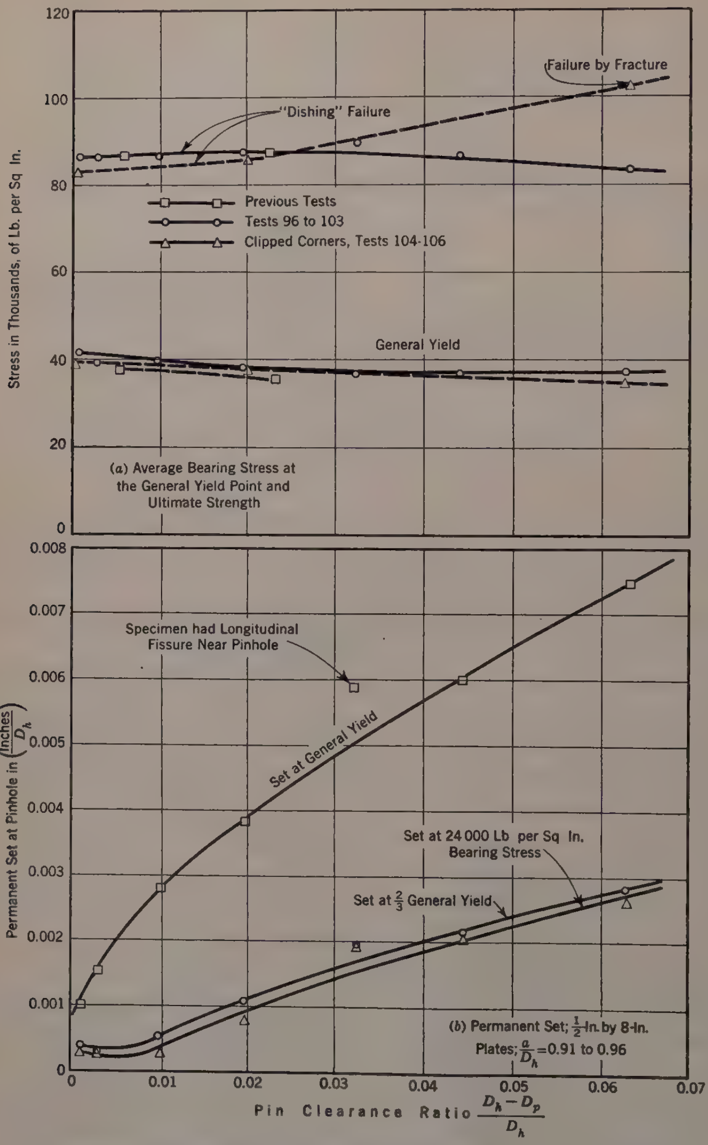


FIG. 19.—EFFECT OF VARIATIONS IN THE PIN-CLEARANCE RATIO, $\frac{c}{D_h} = \frac{D_h - D_p}{D_h}$

RELATION OF TEST RESULTS TO DESIGN

This paper includes no recommendations as to proper working stresses, factors of safety, or allowable permanent sets, which are properly the subject of specifications set up by engineering groups. In the following text, empirical formulas are presented for predicting the average bearing stress at the general yield and ultimate strength of pin-connected plate links similar to those tested. In order to study the four variable dimension ratios which affect the strength of the plates in the regions of greatest variation, many of the test pieces have dimensions outside the usual range of design. As possible design proportions are approached, however, the test results show a uniformity of trend which warrants the application of the formulas to all usual design proportions. Definite limits within which the formulas are applicable are noted in connection with each. The formulas agree closely with the average test results; and, in case of variation of more than a small percentage, they are adjusted to agree with the lowest trend of test results. In applying these formulas to design, these further restrictions are emphasized:

(1) Material should be structural grade steel for bridge work as covered by standard specifications.[8]

(2) Loads are static and pins do not rotate. For repeated stress conditions or for rotating pins appropriate allowances must be made.

(3) No rivet or bolt-holes may be drilled or punched in the vicinity of the pinhole. Previous tests[2] have noted the weakening effect of such small holes, either below or above the pinhole.

(4) The body of the plate shall have the same dimensions as the gross section at the pinhole.

Notation.—Reference should be made to Fig. 1 for the notation in regard to dimensions of plate. Ratios to which reference has already been made are:

$$\frac{t}{D_h} = \frac{\text{Plate thickness}}{\text{Pinhole diameter}}; \quad \frac{b_e}{D_h} = \frac{\text{Edge distance to side of pinhole}}{\text{Pinhole diameter}};$$

$$\frac{a}{D_h} = \frac{\text{Edge distance below pinhole}}{\text{Pinhole diameter}}; \quad \frac{c}{D_h} = \frac{D_h - D_p}{D_h} = \frac{\text{Pin clearance}}{\text{Pinhole diameter}}.$$

The unit stresses are: s_{by} = average bearing stress between pin and plate at general yield of plate; s_{bt} = average bearing stress between pin and plate at ultimate strength of plate; s_y = tensile yield point of material as determined by standard tension test; and s_t = ultimate tensile strength of material as determined by standard tension test.

General Yield Point.—The average bearing stress at the general yield point is not appreciably influenced by plate thickness ratios, $\frac{t}{D_h}$ greater than $\frac{1}{20}$, nor by the edge distance ratio, $\frac{b_e}{D_h}$, for values greater than $\frac{1}{2}$. The test data do not give results for $\frac{b_e}{D_h}$ greater than 1.2, but the bearing stress in such cases at the general yield point is probably not influenced greatly for values of $\frac{b_e}{D_h}$ within the usual range of design. The clearance between pin and pinhole, as defined by the ratio, $\frac{c}{D_h}$, has a minor effect on the general yield point, as indicated in Fig. 19(*a*). The general yield point of the plate is induced principally by a yielding of the material behind the pin. This yielding is preceded by a crushing failure below the pin which progresses until all the metal behind it starts to give way in a plastic state. In very thin plates the general yield is followed quickly by "dishing" and ultimate failure, whereas in thicker plates there is a large plastic reserve before ultimate failure occurs.

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[8] Specification A7-34, Am. Soc. for Testing Materials.

The following formula gives the average bearing stress at the general yield point:

$$s_{by} = \frac{s_y}{2}\left[3\left(\frac{a}{D_h}\right) - \left(\frac{a}{D_h}\right)^2 - 2\left(\frac{c}{D_h}\right)\right] \qquad \text{(1)}$$

Equation (1) applies within the following limits:

$\frac{a}{D_h}$ between 0.3 and 1.2

$\frac{c}{D_h}$ between 0 and 0.07

$\frac{b_e}{D_h}$ greater than 0.5 (otherwise general yield will be entirely in net section)

$\frac{t}{D_h}$ greater than 0.05 (otherwise ultimate failure by "dishing" may precede any yielding)

Equation (1) is based on the test average of s_y equal to 39 kips per sq in. Based on the minimum specification requirement of 33 kips per sq in. for structural steel for plates, shapes, and bars in bridges (A. S. T. M. A7–34) and using ratios of $\frac{a}{D_h} = 1.2$ and $\frac{c}{D_h} = \frac{1}{250}\left(\frac{1}{50}\text{-in. clearance for a pin 5 in. in diameter}\right)$, by Equation (1), s_y is 35.4 kips per sq in. If a factor of safety of 1.5 were suitable with respect to the general yield point, the allowable working stress would be 23.6 kips per sq in., which corresponds closely with the frequently specified allowable stress of 24 kips per sq in. If such a working stress is suitable, it should only be used when the plate is thick enough to prevent ultimate failure at a low load by "dishing." The tests will be examined later to develop criteria of proportions which will insure against "dishing" failure and at the same time give a balanced design.

Ultimate Failure.—The three general types of ultimate failure have been described previously (see heading, "Test Results: Types of Failure"). Three empirical formulas (in close agreement with test results) will be presented to serve as indices of the average bearing stress at the ultimate strength in the case of failure by any of the three types. Applied to a plate of any proportions, the lowest value obtained from the three equations indicates the expected ultimate strength and the type of failure. All the equations are written in terms of average bearing stress between pin and plate, as follows:

Failure by fracture at side of pinhole,

$$s_{bt} = 2s_t\left(\frac{b_e}{D_h}\right) \qquad (2)$$

Failure by fracture below pinhole,

$$s_{bt} = s_t\left[1.13\left(\frac{a}{D_h}\right) + \frac{0.92\left(\frac{b_e}{D_h}\right)}{1 + \left(\frac{b_e}{D_h}\right)}\right] \qquad (3)$$

Failure by "dishing,"

$$s_{bt} = 20 + 315\left(\frac{t}{D_h}\right) + 75\left(\frac{tb_e}{D_h^2}\right) + 20\left(\frac{a}{D_h}\right) - 20\left(\frac{a}{D_h}\right)^2 \qquad (4)$$

In correlating Equations (2) and (3) with the test results, the test coupon average of $s_t = 62$ kips per sq in. applies. In applications to design the minimum specification requirement for s_t should be used. The following limitations as to dimensions are made: $\frac{a}{D_h}$ between 0.3 and 1.2; and, $\frac{c}{D_h}$ between 0 and 0.07.

The first limitation automatically limits the other dimensions (whenever they are determining factors) to values close to the test range. The second limitation indicates the test range but does not enter the equations because the clearance was not found to influence the ultimate strength appreciably. Although $\frac{a}{D_h}$ varied from 0.325 to 1.05 in the tests, the extension to 1.2 is based on the fact that the test results approached a uniform trend in the upper limit of $\frac{a}{D_h}$. Further remarks on each type of failure follow.

Failure by Fracture at Side of Pinhole.—This type of failure occurred only in the 6-in. plates. The test results showed that the material in the net section did not fail until the average stress in that section reached the ultimate tensile strength of the material as determined by a standard tension test. This indicates that the stress concentrations near a circular hole, which exist in the elastic range, have no effect on the ultimate strength of structural steel under static loads.

Equation (2) simply states that if failure is by tension in the net section, the average stress through this section will be the same at failure as in the case of a standard tension test. The stress actually developed, based on original areas and a test average of $s_t = 62$ kips per sq in., was slightly higher than that determined by Equation (2), due probably to the fact that the lower stressed material above and below the minimum section did not allow so much "necking down" and reduction of area as in the case of the straight tensile coupons.

Ultimate Failure Below Pinhole.—A study of the plates that failed in this manner shows that the ultimate strength of the plate is dependent both on the material below and to the side of the pinhole, the results agreeing with Equation (3) for $s_t = 62$ kips per sq in.

Ultimate Failure by "Dishing."—Equation (4) is for plates unsupported laterally at the ends. If the plate is blocked securely against "dishing" at the end, either Equation (2) or Equation (3) may determine the ultimate strength. (Although no blocked plates were tested in the present investigation, this statement is verified by tests made on the pin-plates of the Quebec Bridge (see Table 3)). Equation (4) is not written in terms of s_t, as are Equations (2) and (3), because the "dishing" is primarily a function of stability rather than strength.

TABLE 3.—COMPARISON OF TEST DATA, MAUMEE BRIDGE AND QUEBEC BRIDGE

DIMENSIONS, IN INCHES						RATIOS			BEARING STRESSES, IN KIPS PER SQUARE INCH			
Diameter of hole, D_h	Width of Plate b		Thickness, t	Edge Clearance		$\frac{b_e}{D_h}$	$\frac{a}{D_h}$	$\frac{t}{D_h}$	At failure*	Predicted Average Ultimate Strengths		
	At pin	Body		At sides, b_e	At bottom, a					Side	End	"Dishing"
						(*a*) MAUMEE BRIDGE						
8.00	22.0	10.0	1.375	7.00	7.00	0.875	0.875	0.172	68.6 *D*, *FB*	107.7	*87.8*	*87.8*
8.00	22.0	10.0	1.375	7.00	7.00	0.875	0.875	0.172	61.4 *D*, *FB*	107.7	*87.8*	*87.8*
8.00	22.0	10.0	1.375	7.00	6.00	0.750	0.875	0.172	69.2 *D*, *FB*	92.3	*85.6*	86.1
8.00	22.0	10.0	1.375	7.00	6.00	0.750	0.875	0.172	67.1 *D*, *FB*	92.3	*85.6*	86.1
8.00	22.0	10.0	1.375	8.00	7.00	0.875	1.000	0.172	75.0 *D*, *FB*	107.7	96.6	*85.5*
8.00	22.0	10.0	1.375	8.00	7.00	0.875	1.000	0.172	77.3 *D*, *FB*	107.7	96.6	*85.5*
8.00	22.0	14.0	1.375	8.00	7.00	0.875	1.000	0.172	80.5 *D*, *FB*	107.7	96.6	*85.5*
8.00	22.0	14.0	1.375	8.00	7.00	0.875	1.000	0.172	79.5 *D*, *FB*	107.7	96.6	*85.5*
						(*b*) QUEBEC BRIDGE						
5.03	13.0	13.0	0.375	3.75	4.00	0.790	0.745	0.075	57.9 *D*	97.2	77.4	*51.9*
5.03	13.0	13.0	0.375	3.75	4.00	0.790	0.745	†	77.4 *FB*	97.2	*77.4*	†
5.03	11.0	11.0	0.375	3.75	3.00	0.597	0.745	0.075	52.8 *D*	73.5	73.6	*50.8*
5.03	11.0	11.0	0.375	3.75	3.00	0.597	0.745	0.075	55.6 *D*	73.5	73.6	*50.8*
5.03	9.0	9.0	0.375	3.75	2.00	0.398	0.745	0.075	52.3 *D*	*49.0*	68.2	49.7
5.03	9.0	9.0	0.375	3.75	2.00	0.398	0.745	0.075	55.0 *FE*	*49.0*	68.2	49.7
12.14	28.3	28.3	2.000	12.1	8.08	0.668	1.000	0.165	77.2 *FE*	82.2	92.8	*80.3*
12.14	28.3	28.3	2.000	12.1	8.08	0.668	1.000	0.165	78.4 *FB*	82.2	92.8	*80.3*
12.14	26.3	26.3	1.500	10.5	7.10	0.585	0.865	0.123	70.5 *D*, *FE*	72.0	82.7	*66.6*
12.14	26.3	26.3	1.500	10.5	7.10	0.585	0.865	0.123	72.4 *D*, *FB*	72.0	82.7	*66.6*

* D = "dishing"; F = fracture; B = below pin; and, E = side of pin.

† End of plate blocked to prevent "dishing."

Balanced Design.—Assuming that the plate is thick enough, or that it is blocked laterally, so as to prevent "dishing," a comparison of Equations (2) and (3) yields the following criteria for equal strength or balanced design with respect to the edge distances to the side and below the pin:

$$\frac{a}{b_e} = \frac{1 + 1.75\left(\frac{b_e}{D_h}\right)}{1 + \left(\frac{b_e}{D_h}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Equation (5) gives a slight excess of a distance.

What plate thickness is required to prevent "dishing"? Assuming that the design is balanced in other respects according to Equation (5), the following thickness ratio, obtained from a comparison of Equations (2) and (4), gives the required minimum allowable thickness ratio, $\frac{t}{D_h}$:

$$\frac{t}{D_h} = \frac{-20 + 123\left(\frac{b_e}{D_h}\right) - 20\left(\frac{a}{D_h}\right) + 20\left(\frac{a}{D_h}\right)^2}{315 + 75\left(\frac{b_e}{D_h}\right)} \quad \ldots\ldots\ldots (6)$$

Although Equation (6) gives sufficient thickness to prevent "dishing" in an otherwise balanced design, it also gives more than sufficient thickness for plates which are not balanced according to Equation (5). The "dishing" is more nearly a direct function of the ratio, $\frac{a}{t}$, than $\left(\frac{t}{D_h}\right)$, and Fig. 20 shows the maximum allowable ratio, $\frac{a}{t}$, to prevent "dishing" for plates balanced in design according to Equations (5) and (6).

In general, the tests indicate that the most efficiently designed pin-plate connectors are those narrow in width, thick enough to prevent "dishing," and having material below the pinhole in the proportions specified by Equation (5).

Comparison with Previous Tests.—The results of two previous series of tests on pin-connected plates of larger dimensions than those of the present investigation are presented in Table 3. The second group of ten tests is taken from the report[2] on the Quebec Bridge. Six of these tests were models, using pins 5 in. in diameter, and the remaining four were full-sized hangers with 12-in. pins, and plate widths of 26 in. or 28 in. The results of these tests check well with the strengths predicted by Equations (2) to (4), only two of them falling below, with a maximum difference of about 4%, and the remainder above, by as much as 10% in some instances. The material used in these pin-plates had slightly higher ultimate strength than that used in the present study.

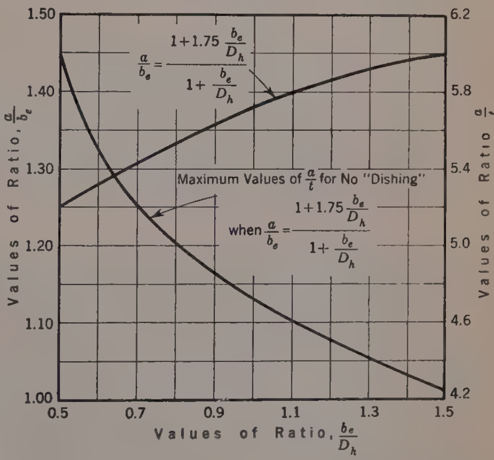

FIG. 20.—BALANCED DESIGN OF ULTIMATE STRENGTH

The results of tests of eight pin-connected plates to determine the best design for anchorage bars for the Maumee River Suspension Bridge are also presented in Table 3. These bars had a reduced width in the main section between pinholes. All the test strengths of these bars fall below the values given by Equation (2), Equation (3), or Equation (4). Apparently, the reduced width in the body of the bar increased the tendency toward "dishing," and reduced their strength, although this may not necessarily be the cause. It is to be noted that the two tests with largest plate width in the body are only 7% less than the strengths indicated by Equation (5). Fracture failures below the pin were reported in these tests but this was combined at the same time, with "dishing." Since the exact dimensions of the specimens are not known and their number is limited, no final conclusion can be given in connection with these tests.

Illustrative Example.—Assume that clearance requirements make it necessary to use a pin-plate 1 in. thick to carry a load of 200 kips at a working stress in bearing of 24 kips per sq in. (A. S. T. M. Specification A7–34). The required pin diameter at 24 kips per sq in. $= \frac{200}{1 \times 24} = 8.33$ in., which dictates the use of a pin with a diameter of 8.5 in. The required net edge-distance at the pinhole, at 18 kips per sq in., is $b_e = \frac{200}{2 \times 1 \times 18} = 5.55$ in. With $b_e = 6$ in., this makes a total width of 20.5 in. required, and the ratio, $\frac{b_e}{D_h} = \frac{6}{8.5} = 0.706$. From Fig. 20, for balanced design: $\frac{a}{D_h} = 0.75 \times 1.31 = 0.985$, from which $a = 0.985 \times 8.5 = 8.4$ in. (use $a = 9$ in.). By actual measurement, $\frac{a}{D_h} = 1.06$; $\frac{t}{D_h} = \frac{1}{8.5} = 0.118$; and, $\frac{a}{t} = \frac{1.06}{0.118} = 9.0$, which is more than the 5.27 allowable maximum for no "dishing," for the ratio, $\frac{a}{D_h} = 1.06$, from Fig. 20. The plate, therefore, will fail by "dishing." By Equation (4), the average bearing stress at ultimate strength is, $s_{bt} = 20 + 315 \times 0.118 + 75 \times 0.118 \times 0.75 + (1.06 - 1.06^2)\,20 = 62.63$ kips per sq in. The ultimate strength is $62.63 \times 8.5 \times 1 = 532$ kips; and, the safety factor with respect to the ultimate $= \frac{532}{200} = 2.66$.

As an alternative, suppose that the plate is securely blocked to prevent "dishing." The ultimate strength will then be given either by Equation (2) for side failure, or by Equation (3) for end failure For $s_t = 60$, the minimum by A. S. T. M. Specification A7–34, Equation (2) gives $s_{bt} = 120 \times 0.75 = 90.0$ kips per sq in.; and, by Equation (3), $s_{bt} = 60 \left[1.13 \times 1.06 + \frac{0.92 \times 0.75}{1 + 0.75} \right] = 96$ kips per sq in. The ultimate strength, then, if the plate is blocked $= 90 \times 8.5 \times 1 = 765$ kips; and the factor of safety with respect to the ultimate $= \frac{765}{200} = 3.82$.

General Yield Point of Plate.—If a pin clearance is $\frac{1}{32}$ in., the pin-clearance ratio, $\frac{b_e}{D_h} = \frac{1}{32 \times 8.5} = 0.00367$. By Equation (1), the average bearing stress at the yield point will be: $s_{by} = 33\left[\frac{3 \times 1.06 - 1.06^2 - 2 \times 0.00367}{2}\right] = 33.8$ kips per sq in.; the total strength at the yield point = 33.8 × 8.5 × 1 = 288 kips; the safety factor with respect to the yield point = 1.44; and the permanent set for a stress of 24 kips per sq in. (from Fig. 19(*b*)) = 0.00025 × 8.5 = 0.002 in.

It should be noted that the general yield point may be raised by arbitrarily increasing the end ratio, $\frac{b_e}{D_h}$, but this will also increase the tendency to "dish." In cases where "dishing" is eliminated by suitable plate thickness or by side blocking, a low factor of safety with respect to the general yield point may be justified, in the opinion of the engineer, due to the high plastic reserve strength prior to ultimate failure.

Clipped Corners.—As indicated by previous tests and a very limited number of the present tests (three in all), the corners of the plate below the pin may be clipped with only slight loss of strength. It is recommended that 20% more net section be maintained on a 45° line through the corner than the minimum at the side or below.

SUMMARY

Within certain well-defined limits as to material and shape, this paper has presented empirical equations for predicting the general yield point, ultimate strength, type of failure, and balanced proportions of pin-connected steel plates. These equations agree closely with the 106 tests reported, in which the pin diameter was 3 in., and are further validated by tests on similar pin-connected links used in the erection of the Quebec Bridge in which the pin diameter was as much as 12 in. Although the results do not check well with a number of tests of pin-connected anchorage bars for the Maumee River Suspension Bridge, this may possibly be accounted for by reason of the reduced width in the body of these bars. Hence, the results of this paper are not intended to apply directly to pin-connected plates having the shape of standard eye-bars unless tests on such eye-bars show agreement with the present results.

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PAPERS

PRACTICAL METHODS OF RE-ZONING URBAN AREAS

By Hugh E. Young,[1] M. Am. Soc. C. E.

Synopsis

The purpose of this paper is to show why re-zoning is a present-day necessity, and to indicate some practical methods of re-zoning urban areas. The first part of the paper is devoted to an explanation of zoning experience in Chicago, Ill. The Illinois State law permitting zoning is outlined, together with the 1923 Chicago zoning ordinance and subsequent amendments in methods and policies, and then a comparison is drawn between land uses as they existed in 1923, the land-use allocations made in the 1923 zoning ordinance, and the land uses as they existed in 1936. The writer believes that this factual demonstration of conditions in Chicago, which are shown to be similar to conditions in other American cities, proves the need for re-zoning.

The latter part of the paper is devoted to the recent development of "measuring sticks" that can be used as guides in the allocation of land for various uses wherever re-zoning is undertaken. A real property inventory and land-use survey are advocated as the initial step. These are to show existing patterns of land use, areas of blight, population changes, and other factors which together make re-zoning necessary. In this part of the paper the technique of re-zoning is outlined, together with the tabulated result of a questionnaire dealing with the need for and method of re-zoning in about seventy-five American cities. The question of blight is dealt with in detail, as also is the matter of public and private housing; and the paper concludes with the writer's recommendation as to what he considers to be a practical method of re-zoning urban areas, particularly with regard to the rehabilitation of blighted districts.

Introduction

The fact that the need for re-zoning urban areas is being discussed means that considerable progress has already been made in zoning. In recent years

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[1] Chf. Engr., Chicago Plan Comm., Chicago, Ill.

city planners have learned much about the technique of proportioning urban use as to amount and location. It has now become desirable to adjust some of the older ordinances to a more "even keel" by means of re-zoning, and to emphasize certain principles that should be followed in the preparation of new zoning ordinances.

Engineers should not be too critical of the work of the planners of fifteen or twenty years ago, because they were pioneers and were trying to correct conditions of urban structure that had been typical for centuries. They had to re-zone men's minds as well as the physical structure of cities.

Need for Zoning

Why Zone?—Many accomplishments in art and science have been achieved and great progress has been made in recent years in architecture, engineering, and public works; and yet city congestion, blight, slums, and misery are still to be found. These by-products of human activity have ever been present as a challenge to the humanitarian instincts of Man throughout the centuries. It is only in comparatively recent years, however, that city planners have come to sense the great social need of transforming the physical structure of urban areas. Zoning is a fundamental and vital element in city planning, because it is the process that co-ordinates the social, economic, and physical structure of urban areas.

Extent of Zoning.—Since 1916, when the City of New York, N. Y., was granted enabling powers by the State Legislature, approximately 1 200 zoning ordinances have been put into effect in the United States, representing approximately 75% of the urban population of the country.

Zoning Handicaps.—As a rule, planning commissions did a creditable zoning job in the early plans, considering the handicaps involved. The art of zoning was then in its pioneer state. Often, it developed that property owners on section line streets (usually the car-line streets, and other thoroughfares) insisted upon having their residential property zoned for business use. Often, too, real estate speculators, through some device or other, obtained concessions favorable to their exploitation process. Many citizens, also, were speculatively minded and hoped, by means of zoning, to have their property transformed from ordinary to high value.

Zoning Experience.—When all the comments, pro and con, have been "put in the hopper," all must agree in the final analysis that zoning has been worth while and that it was valuable during the boom building period prior to 1929, but that it is far from perfect in its present status. The art is still crude with respect to the "tapering-off" process or the transition from a higher to a lower use and the utilization of park strips between residential sections and other uses. Although present zoning plans and ordinances seem to be satisfactory in many smaller cities, officials of most cities, both large and small, feel that re-zoning is necessary. Without doubt, zoning has exercised a beneficial influence in shaping the growth of cities, and often has served as a barrier in preventing abortive property uses.

Zoning Variations.—Variations in residential areas are detrimental because they unstabilize the zoning of the block and of the district; residential protection is destroyed and competition is set up with property already zoned for business. There are innumerable examples in which miscellaneous variations of this character are permitted, and in which penetration of unsuited uses is gradually nullifying the spirit of zoning.

In some instances entire blocks of property are zoned for business where the business structures are in decay; yet only a few blocks away commercial usage has been allowed to penetrate into residential areas. If business had been confined to the area zoned for commercial use, it would have put that property into a revenue-producing position, and would have protected the residential areas, which now are certain to disintegrate, having no stabilization either for residential or business use.

There are several variations or non-conforming uses to be found in nearly every block in Chicago, Ill., for example. It is a common occurrence that if these illegal uses are carried into Court, the cases are continued ten or a dozen times, until the matter is finally dropped.

Zoning Experience in Chicago, Illinois

The administration of a zoning ordinance in a large city is a difficult task. The mayor, aldermen, and all city officials are continually bombarded with requests for variations, which ultimately reach the Board of Appeals. The Chicago Zoning Board of Appeals has shown great fortitude and patience in this job and has also demonstrated a fine understanding of the theory and practice of zoning and the need for re-zoning; but it seems impossible, in the complexity of the situation, to maintain the integrity of the zoning ordinance 100 per cent.

The Chairman of the Board, Mr. James H. Gately, is firmly of the opinion that although it is not present practice in Chicago, a certificate of occupancy is most important for the proper functioning of the zoning ordinance; but such a certificate should not be issued by the same department that is charged with its enforcement.

Protecting Manufacturing Districts.—Mr. Gately is also of the opinion that one of the fundamental safeguards to be provided for in re-zoning is reciprocal protection for manufacturing and residential areas. He feels strongly that there should be a tapering off from manufacturing to commercial to residential use in a manner that will not impose restrictions upon manufacturing development. It has been his experience that residential areas contiguous to manufacturing property are certain to be invaded by commercial use.

The City Council should not permit variations without the prior approval of the Board of Appeals. If there is disagreement, the matter should be referred back to the Board for further consideration.

The Board of Appeals should not have the legal power to break a solid block of use, even if there is a non-conforming use in the block across the street; nor should the Council exercise such authority. Proposed zoning amendments require a two-thirds vote of the Chicago City Council to become effective in the event that a written protest against such amendment is made by 20% of the people on the frontage affected, or of adjacent frontages.

Need for Buffer Organization.—The administration of the zoning ordinance is an almost continuous problem of fighting favors. A strong buffer committee is greatly needed. No variation of any consequence would ever "get by" without political backing. In other words, any proposed variation would be taken care of expeditiously by the routine of the Board of Appeals, and would be properly disposed of, were it not for political interference.

A zoning ordinance will not be effective unless its administration is in strong hands, free from political domination. The essential need is a strong public body, such as a plan commission, to lend support to the Zoning Board of Appeals. In fact, a city plan commission is as essential as a Board of Appeals in preserving the integrity of the zoning ordinance.

Zoning Situation in Chicago.—The main purpose of the Chicago Zoning Ordinance was to protect property devoted to its highest and best use against sporadic invasion of uses which would tend to destroy the taxable value of land and buildings and impair public health, safety, comfort, morals, and welfare. In large measure, however, the ordinance has failed to provide this protection. It was not retroactive and, therefore, could not protect neighborhoods against undesirable developments already in existence.

Visions of enhanced realty values from commercial uses and greater height and volume provisions resulted in public demands for zoning classifications obviously out of line with actual and prospective land and building use. Ordinance provisions permitting additional non-conforming uses were not sufficiently strict to assure districts remaining in the class in which they were zoned. Broad discretionary powers were granted the Zoning Board of Appeals to permit variations in case of practical difficulties or particular hardship.

Chicago Zoning Provisions.—The Chicago Zoning Ordinance provides four "use" and five "volume" districts. The use districts embody residence, apartment, commercial, and manufacturing districts. The volume districts regulate and limit the height and bulk of buildings and the intensity of the use of land, the height limitations ranging from 33 ft in Volume District No. 1 to 264 ft in Volume District No. 5.

Illinois Zoning Law

The Illinois law provides for the establishment of Zoning Boards of Appeal and gives them power to: (1) Interpret the terms of the zoning ordinance or the application as made by the building commissioner; (2) vary the application of the ordinance in special cases of unnecessary hardship; and (3) recommend to the City Council changes in the zoning ordinance.

The State law does not grant Boards of Appeal the power to change a zoning ordinance or to change the classification of property under the zoning plan, but confines the power of amending the ordinance to the City Council.

1934 Amendments.—In 1934, the section of the Chicago Zoning Ordinance of 1923 dealing with the Board of Appeals was amended by the City Council to specify who should be eligible to membership; to provide for board members serving without compensation; and to incorporate certain provisions of the State Act with respect to appeals. In addition, about eighteen variations which the Board might make in specific cases of hardship were enumerated.

The power of permitting variations is in such general terms that it affords the possibility of grave abuse. No standards are defined to guide the decision of the Board, but judgment is left almost entirely to the Board's discretion. For example, the zoning ordinance specifically provides that no manufacturing use shall be permitted in a commercial district. The 1934 amendment authorizes the Board to grant permission to devote premises in a commercial district to a manufacturing use where the appropriate use of neighboring property is not injured thereby.

Variations of this character may be right and necessary to prevent hardship, even if they are in direct conflict with the language and purpose of the zoning ordinance; but there is a strong possibility of abuse by unreliable Boards when the Board constitutes the sole judge as to whether or not the non-conforming use is injurious to neighboring property. In the hands of an undependable Board the variations permitted may easily amount to such a volume of precedent as practically to nullify the purpose and intent of the entire ordinance.

Present Use Factor.—In developing the need for re-zoning, the most practicable "measuring stick" appears to be the factor of use. The research already made in this direction is of tremendous importance. Significant comparisons can be made based upon conditions in Chicago.

Fig. 1(*a*) shows the use of property in Chicago as of 1923 when the zoning ordinance was passed. At that time, the incorporated area was 201 sq miles and comprised the uses shown in Table 1(*a*). The developed area comprised 143 sq miles. The remainder of the area (58 sq miles) was occupied by water, or was vacant.

TABLE 1.—LAND USE IN CHICAGO, ILLINOIS

Uses	(*a*) As of 1923		(*b*) As Zoned in 1923		(*c*) Effect of Suburban Factors	
	Area, in square miles	Percentage of incorporated area	Area, in square miles	Percentage of incorporated area	Area, in square miles	Percentage of developed area
Single-family dwellings	23.75	11.8	5.91	2.9	62.25	26.6
Two-family dwellings	15.66	7.8	38.55	19.1	18.70	8.0
Multiple-family dwellings	9.19	4.6	26.00	13.1	15.98	6.8
Total dwelling areas	....	24.2	....	35.1	....	41.4
Commercial areas	9.87	4.9	28.24	14.0	13.61	5.8
Manufacturing areas	25.91	12.9	48.64	24.1	34.45	14.7
Special areas	4.95	2.4	....	....	11.92	5.1
Streets and alleys	46.60	23.3	46.60	23.3	69.30	29.6
Parks and playgrounds	7.06	3.5	7.06	3.5	8.06	3.4
Water and vacant property	58.01	28.8	....	....	....	....
Total	201.00	100.0	201.00	100.0	234.27	100.0

Fig. 1(*b*) shows the manner in which the area was zoned in the 1923 ordinance. The entire area of 201 sq miles was zoned for the uses indicated in Table 1(*b*). All vacant areas were defined for use of some character. Water areas were zoned for the same use as contiguous areas. A comparison of Figs. 1(*a*) and 1(*b*) shows that, although the zoning ordinance increased the total dwelling areas 1.45 times, the single-family dwelling areas were decreased to less than one-fourth the space then in actual use. Commercial areas were increased 2.86 times, and manufacturing areas were increased 1.88 times the space then in actual use.

Actual and Zoned Use Situation in 1936.—At the close of 1936, there had been little actual change in land use despite the great increase it was originally assumed would take place in manufacturing and industry. In the last thirteen years of that period 100 539 building permits were issued, which, translated into land use, is shown in Fig. 1(*c*). During that period an estimated total of 21 965 permits was issued for demolition of property, which number, translated into shrinkage of use, shows a decrement of 3.15 in percentage of developed area. The net percentage of added developed area occupied during the 13-yr period for dwelling purposes was 8.0; for commercial areas, 1.9; and for manufacturing areas, 0.46.

Number of Variations.—For this same period a number of variations occurred in the zoned use, the net interchange in the developed area (see Fig. 1(*f*)) being: An increase of 0.55% for total dwelling areas; an increase of 0.60% for commercial areas; and a decrease of 0.21% for manufacturing areas.

The graphs in Fig. 1 are of interest in that they show little agreement with the absorption of use as predicted in the 1923 ordinance. There was little change in the areas actually devoted to commercial and manufacturing use, which uses were provided for, generously, in the original ordinance.

Residential vs. Business Use.—Residential use, which was assigned the least area for expansion, showed the greatest development in the period. Demolition has not helped the situation because the largest areas thrown into disuse are commercial and manufacturing, and those are the areas that were assigned the greatest percentage of use in the 1923 ordinance. The trends are somewhat irrational, of course, because of the interrupted normal use absorption resulting from the depression years from 1929 to 1935. Considerable recession has occurred in the use of railroad and manufacturing property.

The Situation as a Whole

The percentage of developed area occupied by various land uses applies only to the area within the city limits, which does not, of course, give the "true picture" for determining the land use factors for Chicago as an entity. In order to have a proper comparison with other self-contained cities, the data should include the suburbs, because they are largely residential areas for the workers of Chicago.

Fig. 2(*b*) shows the developed area of Chicago in 1936 within the city limits, amounting to 157.77 sq miles, and Fig. 2(*a*), the developed residential areas beyond the city limits, comprising 76.50 sq miles.

Suburban Factors.—Only those suburbs have been included in this study that are integrally a part of Chicago even if located outside the corporate limits. Indicating the use for "Greater Chicago," as shown in Fig. 2, the present land uses, in percentage of developed area occupied, are as listed in Table 1(*c*).

Comparison with Independent Survey.—In Fig. 2(*d*) is shown the corresponding percentage of developed area occupied by various uses in sixteen self-

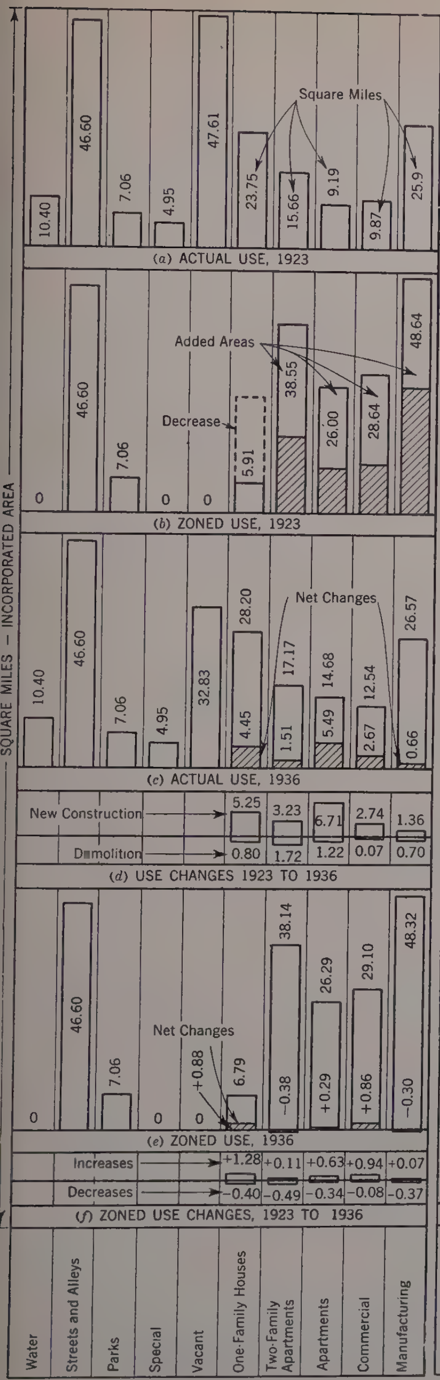

Fig. 1.—Land Use in Chicago, Illinois

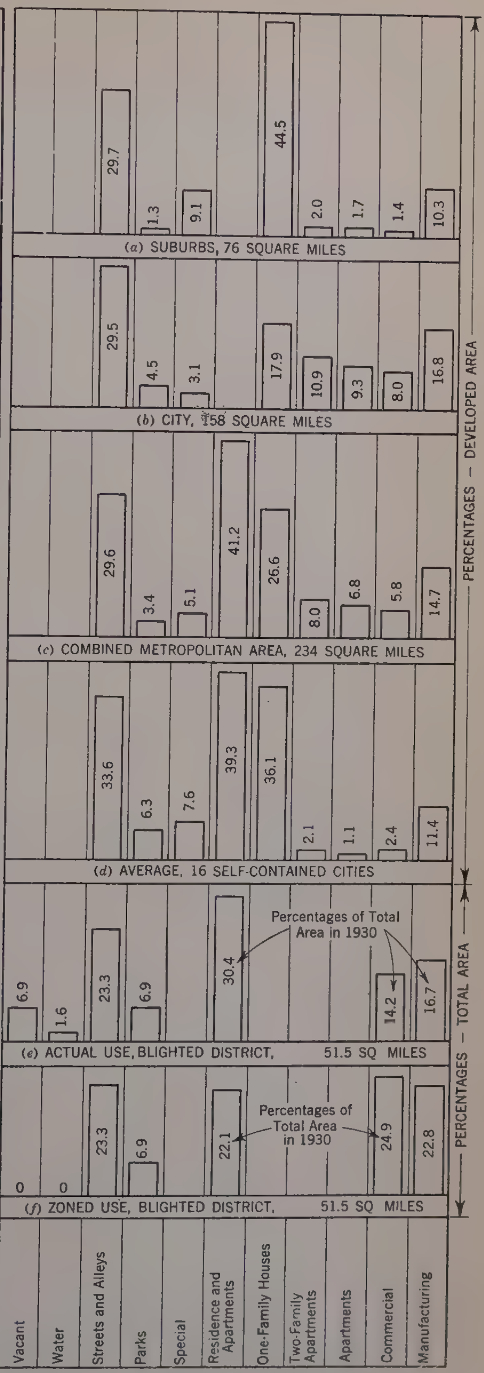

Fig. 2.—Percentages of Developed Area, Chicago, Illinois

contained cities as disclosed in a survey made by Harland Bartholomew, M. Am Soc. C. E.[2] It will be noted that the percentages of occupied developed area in these sixteen cities and in Chicago are in very close agreement (see Fig. 2(*c*)). This indicates that factors determined by his survey are applicable to large urban areas, such as Chicago, with the exception of the classification of residential use. In Chicago, and probably in other similar cities, a much greater percentage must be allowed for two-family and apartment house use, and a shading of use for single-family dwellings. The ratio of all land-use areas to population, in terms of acres per 100 persons, can readily be determined from these percentages.

When the information developed by land-use surveys of other large urban centers has been compiled in terms of percentage of area developed for the various uses, city planners will have a valuable means of allocating city area, definitely, for various uses and for different classifications of city population. Therefore, any city, after it has reasonably determined its prospective future population, can zone properly for present use and can determine the proper proportion of future use absorption.

Zoning Mistakes.—When the present use of property in Chicago is compared with the zoned use, and it is found that after thirteen years practically none of the increased use anticipated in 1923 has occurred, one cannot escape the conclusion that present zoning methods are badly distorted and that Chicago must be re-zoned.

Zoning in Chicago is not only distorted with respect to an illogical allocation of usage, but also with respect to the distribution of various uses. Little consideration has been given to tapering off from highly restricted uses to unrestricted uses. There is a lack of buffer property between manufacturing use and residential use.

Chicago (and perhaps other cities as well) arbitrarily zoned major and minor streets in checker-board fashion for commercial use, and zoned areas for residential use contiguous to manufacturing use. It will be necessary to delete miles of such commercial zoning and to replace it with residential use. Furthermore, it will be necessary to re-zone areas contiguous to manufacturing property from residential to commercial use.

"Blue-Sky" Zoning.—"Blue-sky" zoning has complicated the tax machinery. If valuations are based on the present value of hundreds of miles of fictitiously zoned commercial property, where because of its blighted nature there is no income, this cannot possibly mean anything except tax delinquencies. This also applies to many areas that have been zoned for large volume residential property. The deficit thus produced must be made up in some other manner.

Surveys in Cleveland, Ohio, New York, N. Y., Boston, Mass., Los Angeles, Calif., and other cities give volumes of information on this subject. As a matter of fact, improper zoning is a blight in itself, in that it specifies a use and volume that can never be attained. While awaiting the advent of such expected use the property is often allowed to fall into disuse because it does not seem worth while to spend the money to keep it in condition for its former use.

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[2] "Urban Land Uses," by Harland Bartholomew.

Re-zoning beyond the urban limits is also necessary, if for no other reason than the need of safety. The removal of the "honky-tonk," mushroom growth of "hodge-podge" commercial uses that now clutter the main highways and create a dangerous marginal friction, can be accomplished by pushing these uses, where needed for the local community, back of the highways in a manner similar to that being done on highways in some foreign countries.

Re-Zoning Needs.—One notable example in Chicago of the impropriety of zoning residential areas near manufacturing areas is the Trumbull Park Homes Project of the Public Works Administration (PWA) in the South Chicago industrial district. This is an area of 21.5 acres, the approximate boundaries of which are Trumbull Park, Bensley Avenue, and the Chicago and Western Indiana Railroad. The housing project is located in the very heart of a large manufacturing area. The Government asked for a revision in the zoning ordinance, but when it was discovered that if this area were re-zoned for residential use it would immediately render all industrial property within 2 000 ft of it useless for certain manufacturing purposes similar to those now there, the request for this zoning revision was withdrawn, although the housing project was continued.

Manufacturing property needs protection just as much as residential property, and this protection would seem to be the utilization of contiguous areas for commercial use, tapering off into residential use, the latter being removed far enough from manufacturing areas so as to avoid restricting the free use of such manufacturing areas. There is also opportunity in this buffering process for an appropriate interposition of needed park areas between residential and commercial use.

Comparison with a Hotel.—A city is a corporate entity that should be as susceptible of the intelligent application of economic principles as any business corporation. Consider a hotel, for example, and apply to it the factors of Chicago's zoning: 52% of the hotel's floor area was zoned for show rooms, restaurants, and other commercial uses, including its operating facilities, whereas only 39% of the total floor area was being used for these purposes; only 4% was zoned for single rooms, whereas 29% was being used for this purpose; 26% was set aside for double rooms and only 17% of the area was actually being so used; and 18% was zoned for suites whereas only 15% of the normal hotel area was actually being so used. Every one must appreciate how illogical such a condition is when applied to a business corporation; but few people seem to realize that it is equally illogical when applied to a municipal corporation. The proper basis for zoning is the present use factor, and anticipated future absorption of vacant property must be provided for on the basis of this factor.

BLIGHT

Description.—In the larger cities, at least, re-zoning is essential before blighted areas can be rehabilitated. This is the situation in Chicago, where the blighted district is twice the size of Manhattan Island, New York. It is the backyard of the lake-front development, and stretches virtually from Belmont Avenue on the north to 63d Street on the south, and from the "Loop" west to Kedzie Avenue. This comprises a total area of 51 sq miles, of which 44 sq miles are blighted territory. In this district, which is about one-fourth the total area of all Chicago, live two-fifths of the city's population—about 1 300 000 people. The district contains 169 000 families of the lowest income levels. Maps of this district, prepared by the Chicago Plan Commission in collaboration with the Social Science Department of the University of Chicago, show that the density of population in the 287 census tracts in the district are as given in Table 2.

TABLE 2.—DENSITY OF POPULATION, CHICAGO, ILLINOIS

Population density in persons per acre	Number of census tracts	Percentage of total
Less than 30	23	8.0
30 to 60	106	37.0
60 to 90	111	38.7
90 to 120	43	15.0
120 to 150	4	1.3
Total	287	100.0

Use Data.—Comparing the 1930 use map of this area with the zoned use, the condition is as shown in Figs. 2(*e*) and 2(*f*). The effect was to decrease the population in total dwelling areas, 27%; increase it in commercial areas, 75%; and increase it in manufacturing areas, 37 per cent. These data certainly are a strong argument for re-zoning this area. The need for rehabilitation of the area is emphasized by the fact that 70% of the district is occupied by frame structures from 30 to 60 yr old.

Home Ownership.—To what extent do the residents of this section own the homes in which they live? In the 414 census tracts of the 1930 Census the distribution is as shown in Table 3. In 72 tracts less than 10% of the families own their own homes.

TABLE 3.—HOME OWNERSHIP

EXTENT OF HOME OWNERSHIP; RANGE OF PERCENTAGES		Number of tracts	Percentage of total tracts
From:	To:		
	<1	4	1.0
1	9	68	16.5
10	19	94	22.7
20	29	123	29.7
30	39	75	18.1
40	49	31	7.5
50	59	11	2.6
60	69	8	1.9
Total	....	414	100.0

TABLE 4.—RENTALS

RANGE OF AVERAGE MONTHLY RENTALS		Number of tracts	Percentage of total tracts
From:	To:		
*	*	4	1.0
$10	$19	37	9.0
20	29	140	34.0
30	39	97	23.5
40	49	87	21.0
50	59	22	5.2
60	69	18	4.2
70	>70	9	2.1
Total	....	414	100.0

* There are no homes in these tracts.

Rentals.—In the 414 census tracts of the 1930 Census, the median rental values were as shown in Table 4. The living quarters in one-third of the census tracts in the district yield an average monthly rental of $40, or more, and the living quarters in two-thirds of the tracts yield only from $10 to $39 per month.

The zoning volume regulations in this district are such that a building height of 66 ft at the street line and 132 ft maximum with set-backs, may prevail in more than one-half the area.

Population Changes.—One of the effects of blight and an indication of its presence, is the mass movement of residents away from a district. The boundaries of the territory of principal population decline of Chicago, 1920–1930, shown by the broad black band on Fig. 3, coincide generally with the boundaries of the blighted district. This district contained 1 060 716 residents in 1920. In the 10-yr period it lost 146 354 people, or 13.8%, leaving 914 362 people in 1930.[3] In the next four years the decline continued, the population dropping to 848 803. The total decline in this area for the 14-yr period amounted to 211 913, or nearly 20 per cent. In 1934, the boundaries of the territory of principal population decline had moved outward as shown by the broad black line on Fig. 4. The 1920–1930 territory of decline is shown bounded by a heavy broken line. The territory between these two lines contained 1 633 596 people in 1930. In 1934, this number had declined to 1 563 505, a loss of 70 091 residents, or 4.3 per cent. The 4-yr loss of the original district shows a population decline of 282 004 for both areas. Until the years 1920 and 1930, respectively, these areas had shown a steady increase in population.

Rehabilitation of Blighted Areas.—Blighted areas in Chicago must be re-zoned regardless of any particular method that may be adopted for financing their reconstruction. Some of the sources of blight are the following: (1) Decentralization and moving away of factories and industries; (2) public improvements that provide outlying developments which induce the higher paid residents of a district to abandon the old and move out to the new; (3) faster transportation that enables people to live farther away from their place of employment; (4) retention of old unfit housing and lack of suitable housing facilities for lower income groups, resulting in over-crowding of run-down districts; (5) single-family dwellings occupied by a multiplicity of families; (6) removal of the established residents and the incoming of new residents with less civic interest and generally with less income; and (7) depreciated land values; weak owners; repeated change in ownership; lack of adequate maintenance because of surrounding dilapidation and fear of what adjoining property owners might do that would still further depreciate values and income.

Annual Deficits.—If city expense in Chicago were pro-rated on the basis of area and compared with revenues received from the blighted districts, the results would be a deficit of tens of millions of dollars, assuming that all taxes were paid. As a matter of fact, there are 24% delinquencies in tax payments, so that if the deficit was paid by 10% of the city area, including the high-class business and residential property, as was shown to be the case by a survey made in Boston, Mass., that fact should be some incentive for the city's leaders of industry, business, and commerce, to devote more attention to the rehabilitation of blighted areas.

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[3] "Census Data of the City of Chicago," 1920 and 1930 Rept., Social Science Dept., Univ. of Chicago.

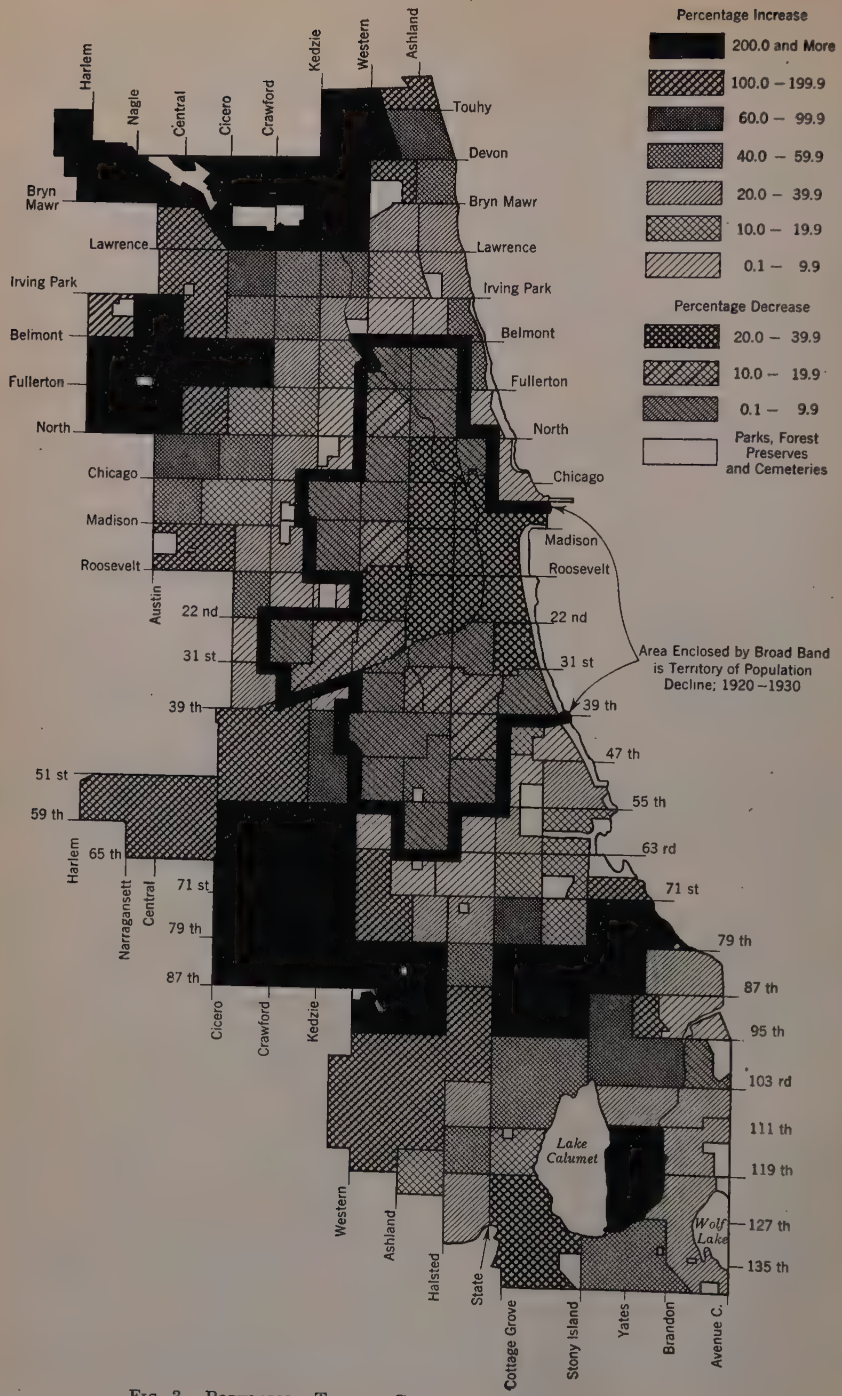

FIG. 3.—POPULATION TRENDS, CHICAGO, ILLINOIS, 1920 TO 1930

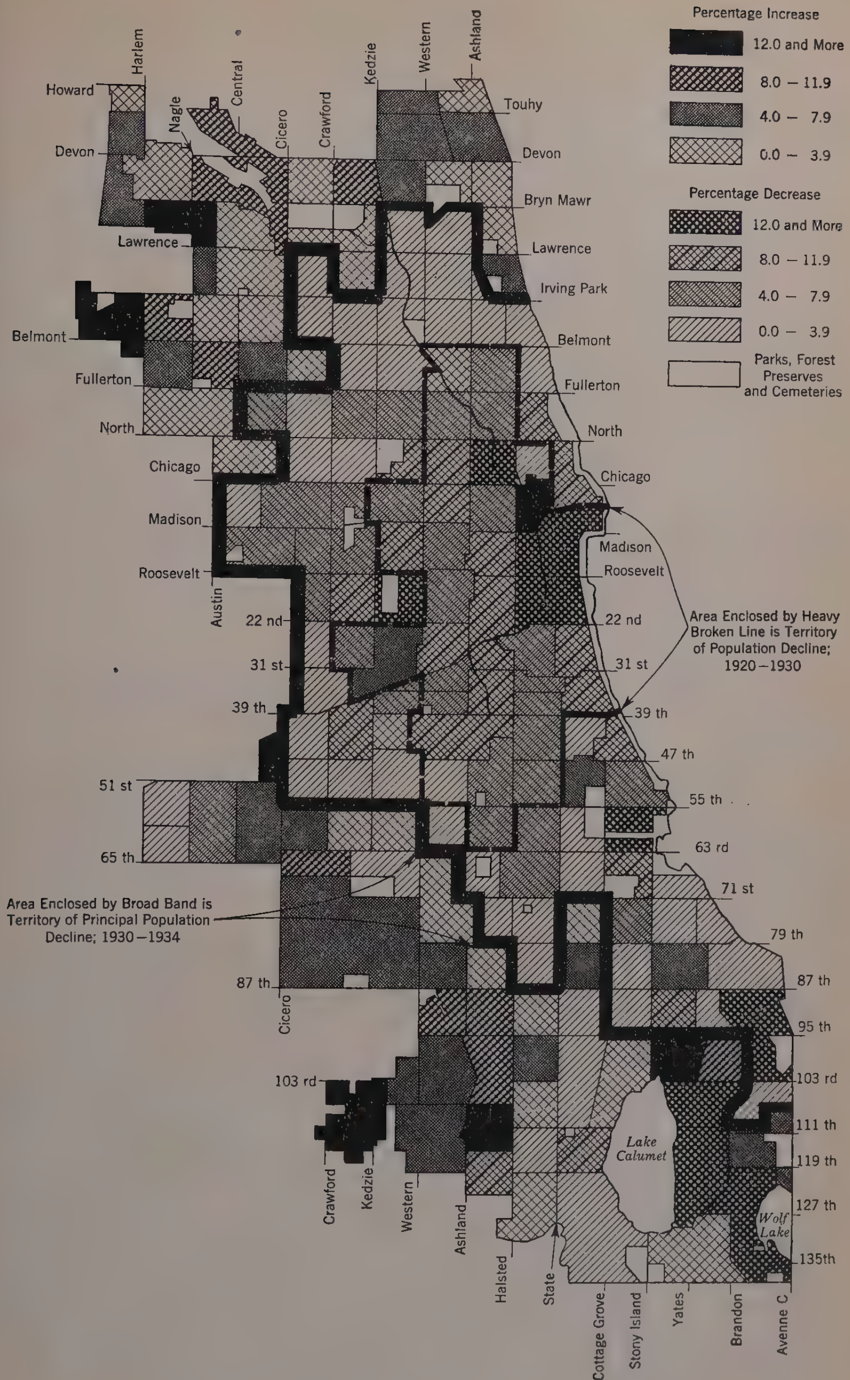

Fig. 4.—Population Trends, Chicago, Illinois, 1930 to 1934

The question is always raised: "How can water be made to run up hill?" The answer is to be found in two fundamental factors: First, vital consideration of the sources of workers' income; and, second, the ability of workers to pay rentals for living quarters of not less than "low-cost" housing standards.

Manufacturing Removals.—For years there has been a recession of manufacturing use and the contingent sources of labor, such as railroad service, etc., in the areas adjacent to the "Loop," in Chicago. The West Central Association in Chicago stated in 1936 that:

> "$30 000 000 is a conservative estimate of the amount of business which is driven away every year by the lack of good transportation arteries, the disintegration of neighborhood areas and other causes which have brought about the dry rot which has been steadily eating into the entire west side section during the last twenty years. One business concern after another is forced to move into other districts where more favorable conditions are available. In the five years between 1924 and 1929, seventy-five large industries moved from the west side to the outskirts of the city."

Manufacturing must be encouraged to remain in blighted areas. These plants must be given protection against such zoning of contiguous areas as will restrict manufacturing expansion. New manufacturing concerns must be induced to locate in blighted districts because this is an essential means of stabilizing these fast deteriorating areas. Adjustments must be made in tax valuations to bring tax rates in blighted districts down to a level as favorable as that in effect on the outskirts of the urban areas.

Income Needs.—People cannot live in any kind of housing conditions without an income. A. C. Comey, M. Am. Soc. C. E., has described the situation pointedly:

> "A slightly different approach to population planning is afforded by combining it directly with a study of activities, largely expressed in terms of numbers of people. Thus we may inquire not merely where people have been, are, and are likely to be, but also why. * * * Pursuit of livelihood is a most compelling reason "why," for most of the population."

The problem of a standard minimum wage is a national one, but it is evident that the income of a family gainfully employed cannot be less than, say, five times the rental of "low-cost" housing. Rentals, of course, will vary in different sections of the country.

There are two important factors involved, therefore, in the rehabilitation of blighted areas. In some manner, present manufacturing will need to be induced to remain in blighted areas, and should be given opportunity to expand its plant by proper zoning; additional diversified manufacturing and industry will have to be induced to locate in these districts; and it is evident that the income of a family gainfully employed must be such as to permit it to live in dwellings of not less than standard low-cost housing.

The rehabilitation of blighted areas has been an enigma for many years, and it will continue to be an insoluble problem unless these two factors are properly determined, although they seem very commonplace in the array of panaceas that have been advanced for social betterment.

What Do the Studies Show Further?—Referring once more to the studies made, it is interesting to determine what they show with respect to blight and the need for re-zoning. They show that:

(*a*) In Chicago there are 44 sq miles of blighted territory piling up enormous deficits every year, which deficits must be met by the owners of property in other areas. The owners of property in this area are paying a very small share of taxes annually because they have allowed their property to depreciate and do not keep it in a condition that would give it adequate value to bear its proportionate share of city expense.

(*b*) These blighted areas may have had some prospect of development before zoning, but their prospects are now stationary because they are improperly zoned. They must be re-zoned; and there must be enforcement of demolition orders when owners refuse to make repairs, renewals, etc., necessary to make their property habitable in accordance with minimum living standards.

(*c*) The zoned use of these areas is entirely out of line with the use shown to be possible by the application of normal use absorption factors.

(*d*) There are miles of spurious commercial frontage and the pattern of location is irrational. Fortunately, little has happened in Chicago to change the "picture" since 1923. Hence, Chicago can be re-zoned in accordance with present standards without changing anything more than the zoning plats.

A Practical Method of Re-Zoning

When it comes to a practical method of re-zoning it should not be necessary to argue in behalf of the necessity of a land-use survey as a first step. Sixty-four surveys have been made in the United States and, no doubt, many more are under way. Most city planners are familiar with the startling disclosures made by these surveys.

If comprehensive land-use surveys cannot be financed, reasonably good results in fact-finding can usually be obtained from census data, use of insurance plats, assessors' cards, and other data on file in city departments.

Re-Zoning Plan.—The second step, of course, is the preparation of the official re-zoning plan based on fundamental use factors, determined from the survey and thoroughly diagnosed with respect to the average use factors of other cities. Officials have carefully estimated the future population growth of all cities so that, by applying the ratio of use per 100 population, a final total percentage of developed area can be arrived at for each use. Some cities probably have sufficient vacant property to accommodate future requirements. If not, the application of the use factors will give them a good idea of the additional area that should be incorporated. The process then becomes one of intelligent allocation of proportioned use areas.

Manufacturing Areas.—The location of manufacturing areas will be largely determined by transportation facilities. The zoned areas for this use should be checked off, and increased or decreased in size according to the use factors, but not changed in location, if the original zoning was logical. In Chicago, the location of the manufacturing areas would not be substantially changed in any re-zoning plan.

Commercial Areas.—Of course, all existing important commercial centers, areas, and frontage of major street intersections will be included as routine re-zoning. Vacant commercial frontage should be eliminated from the area zoned for commercial uses unless shown to be needed by the use absorption factor.

In allocating surplus area for future expansion of business, two important factors are involved: (1) Commercial use should be interposed between manufacturing use and residential use; and (2) proper provision should be made for proportioning the expansion of present live commercial areas. The foregoing process should proceed until the total anticipated use is absorbed.

Apartment Areas.—Multiple-family residential use should be zoned for major and minor arterial frontage. In the process of rehabilitating the blighted district these thoroughfares should receive appropriate design and cross-section development as part of the reconstruction project.

Residential Areas.—Two-family and single-family dwellings in general should occupy areas buffered by multiple-family use, parks, and commercial areas.

In examining the map of Chicago's blighted district, one may see how these factors can be applied in re-zoning the area without upsetting anything more than the 13-yr-old pages of the original zoning ordinance. Nothing much has happened since 1923 to change the "picture."

The next step, of course, is the publication of the proposed re-zoning plan, with a complete disclosure of the social, economic, and physical conditions involved and the advantages to be gained by the proposed re-zoning. That should be followed by a campaign to inform the citizens as to the benefits to be derived from re-zoning.

Summary of Answers to Questionnaire.—On December 18, 1936, the writer circulated questionnaires to 154 cities, and 102 replies were received from cities having a total population of 26 689 000, or more than 22% of the entire 120 000 000 population of the United States. The purpose of the questionnaire was to reveal the zoning situation as it exists in cities throughout the nation. This is adequate to give a true cross-section of the zoning situation throughout the entire United States. The data developed by the questionnaire are shown in Table 5.

Of the 102 cities that replied, 88 are zoned and 14 are not zoned; of the 88 zoned cities, 59 declare that re-zoning is necessary and 29 state that it is not necessary in their particular cases. Of these 88 cities, 9 have been re-zoned, according to the replies, whereas 37 others have been partly re-zoned, a total of more than one-half the entire number.

Of the 77 cities with less than 250 000 population, 39 feel the need for re-zoning, and 27 do not. Ten cities did not express an opinion one way or the other. In cities of this class the sentiment seems fairly evenly divided. In the 25 cities of 250 000 population and more, 20 report that re-zoning is needed, 2 that it is not needed, and 3 did not reply. Thus, it seems clear that the need for re-zoning is much more evident in cities of more than 250 000 population. Of these 25 larger cities, two have already been re-zoned and 10 others have been partly re-zoned. The purpose of the questionnaire was to determine the best method of gaining public approval for re-zoning. The question was propounded in the form of four alternatives.

Public Approval for Re-Zoning.—Question 4 asked if public approval of re-zoning should be based entirely upon the general advantages to be derived; that is, is re-zoning desirable?

TABLE 5.—RESULTS OF QUESTIONNAIRE TO DETERMINE THE ZONING SITUATION

CITIES GROUPED ACCORDING TO POPULATION (IN THOUSANDS)		No. of questionnaires mailed	No. of replies received	(a) NEED FOR RE-ZONING						(b) METHOD OF RE-ZONING										
From	To			No. of cities zoned	No. of cities not zoned	Need for Re-Zoning		No. of cities re-zoned	No. of cities partly re-zoned	Question 4		Question 4(a)			Question 4(b)			Question 4(c)		
						Yes	No			Yes	No	First choice	Second choice	Third choice	First choice	Second choice	Third choice	First choice	Second choice	Third choice
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)
50	100	74	47	36	9	24	17	4	21	40	0	36	0	2	5	12	16	7	15	10
100	250	47	30	27	3	15	10	3	6	20	0	22	1	1	3	6	11	3	12	3
250	500	22	15	14	1	12	1	2	6	13	0	13	0	0	0	5	3	0	3	5
500	1 000	8	7	7	0	6	1	0	2	6	0	6	2	1	1	1	3	2	2	1
1 000	>1 000	3	3	2	1	2	0	0	2	3	0	3	0	0	0	0	0	0	0	0
Total	....	154	102	88	14	59	29	9	37	82	0	80	3	4	9	24	33	12	32	19

Eighty-two cities voted "yes" on this question. No city voted "no." Thus, in 100% of the cities replying, the correspondents felt that after the re-zoning plan had been completed as outlined, public and official approval should be sought on the basis of the benefits disclosed by it, with property owners being given the opportunity to voice objections, so that provision could be made for special conditions.

Re-Zoning for Highest and Best Use.—Question 4(*a*) asked if public approval of re-zoning should be based on the argument that by reducing the area overzoned for commerce, industry, and apartment use, the value of the areas properly zoned for these purposes would be increased as a result of such limitation; that is, that the smaller area for each use would increase land values.

Replies to this question were received from 87 cities. Of this number 80, or 92%, chose this method as being preferable to the remaining two alternative methods suggested in the questionnaire. Three cities picked Method 4(*a*) as their second choice; and four others picked it as their third choice.

Among cities of less than 250 000 population, 93% of those answering the questionnaire selected Method 4(*a*) as being the best; 5% of the cities selected it as their third choice; and 2% selected it as a second choice.

Upon the adoption of an official re-zoning plan there would be a logical basis for the establishment of valuations for taxation purposes, that would be consistent with the actual development of an income from property. This is a condition that is now impossible in large sections of the United States. To-day, all property is menaced because of lack of stability. In the long run, even without intensive rehousing, the corporate fund will benefit if property can be placed in its proper class of highest and best use for development, and will be stabilized through the logical forces of use requirements which operate gradually to force development, following the law of supply and demand. For this to be effective, however, variations (the "worm" that is now destroying the "fabric" of zoned use) must be exterminated by a strong city planning and housing council.

Paternalistic Method of Re-Zoning.—Question 4(*b*) asked if public approval of re-zoning should be sought because of the value of rehabilitating blighted districts through governmental financing; that is, by paternalistic methods.

Replies were received from 66 cities. Of these, 9 selected it as their first choice; 24 selected it as second choice; and 33 selected it as third choice. Thus, only 14% considered it to be of first importance, 36% of second importance, and 50% of third importance. Among cities of less than 250 000 population, 15% considered Method 4(*b*) to be the best; 34% ranked it as their second choice; and 51% felt it to be their third choice. Thus, the smaller cities clearly feel Method 4(*b*) to be the least desirable of the three alternate methods suggested. Among cities of 250 000 population and more the vote on Method 4(*b*) *versus* Method 4(*c*) for second and third place, respectively, was fairly even.

Non-Paternalistic Method of Re-Zoning.—Question 4(*c*) asked if public approval of re-zoning should be sought because of the value of rehabilitating blighted districts through non-paternalistic methods, such as some form of trustee plan, limited dividend corporation, public utility housing corporation, or other private organization, aided by governmental financing, but not under governmental control.

Sixty-three cities replied to this question. Of this number, 12 cities, or 19%, selected Method 4(*c*) as their first choice; and 32 cities, or 51%, selected it as their second choice. Thus, among all the cities voting, the majority selected Method 4(*c*) as representing their second choice among the three alternative methods.

The Housing Situation.—Summarizing the general housing situation one finds that:

(*a*) The Reconstruction Finance Corporation (RFC) was established in 1932, with power to issue building loans for limited dividend housing projects subject to State or municipal housing approval.

(*b*) The Public Works Administration (PWA) began its housing program in 1933 by making loans to limited dividend housing corporations initiated by the RFC. Seven projects were financed in this way. The PWA turned to Federal construction, under which plan fifty projects were undertaken.

(*c*) Since December, 1934, 20 States have enacted housing legislation in accordance with drafts submitted by the PWA, providing for the creation of housing authorities that have broad powers to undertake and operate housing projects, but which are without taxing power. The authorities are given the right to secure property by eminent domain.

(*d*) Now (1938) the PWA is decentralizing public housing and turning over to the State and local housing authorities the responsibility of initiating, constructing, and operating low-rent housing projects.

Housing Needs.—The following items should be kept in mind in determining the problems of the State and local authorities in attempting to initiate, construct, and operate low-rent housing projects: (1) The enactment of enabling legislation; (2) it is necessary for the municipality to furnish the housing authority with sufficient money to defray its first year's administrative and overhead expense; (3) municipalities should be empowered to grant various forms of assistance to the housing authority, in addition to tax exemptions; (4) municipalities might find it necessary or expedient to sell, lease, or donate property to the authority to furnish parks, playgrounds, and other facilities, and to re-zone or re-plan affected areas; (5) it will frequently be necessary in connection with the financing of housing projects for long-term agreements to be entered into between the housing authority and the municipality regarding the furnishing of services and payments therefor in lieu of taxes; (6) to accomplish Objectives (1) to (5), the PWA is suggesting the enactment of the housing corporation law; (7) the PWA will operate fifty housing projects; (8) it is suggested that under appropriate and effective State legislation these projects be leased or sold to local housing authorities as soon as possible; and (9) it is also suggested that, in the future, all low-rent housing projects should, in general, be undertaken by local housing authorities on a loan-and-grant basis.

The proposed plan apparently obviates some of the difficulties of the former plans for financing low-cost housing, in that the municipal housing authority is given the right to acquire property by eminent domain.

Modification of Housing Plan.—Would it not be well to consider some modification of present and proposed legislation whereby property owners and mutual interests could assemble their property into an equity or community trust and construct, own, and administer low-cost housing properties with the aid of a Government loan and grant, subject to all the requirements to be imposed by the municipal housing authority under the present arrangement? Such a plan, of course, should be based upon a limited return on the investment of the property owners. From a purely social point of view this would be much better than the present condition in which slums are subsidized by draining municipal treasuries.

A co-operative trustee plan through this kind of financing would make possible the development of much greater areas for low-cost housing than is possible under the present plan, because of the pooling of the property itself in the development. The municipality would be much better off in the long run, because it would receive revenue from a much larger area than is possible under the present plan, as rehabilitation could be made much more extensive.

Criticism of the plan because it would involve a Federal grant to a co-operative society for private profit should take into consideration that the subsidy would be for a public purpose. It has certain "strings" attached to it in that the equity trust must be operated for limited profits and must achieve low rentals consistent with those necessary to reach the levels of the standard low-cost housing. The trustee plan would employ labor and stimulate industry on a much larger scale than would be possible with the same expenditure of funds under the housing authority.

There is objection to the Government housing authority as the only means of developing housing because it weakens private initiative. This is avoided in the equity trust plan as the property rights and control would remain with the individual owners. Maintenance and operation would be more efficiently administered under the equity trust than under political control; taxation problems would be clarified, and the public would be free from the ever-growing expense of amortization due to obsolescence.

Prime requisites for this kind of plan would be re-zoning, and the selection of cities where manufacturing use exists to give employment to occupants of housing projects.

Some Opinions

Many people are not in favor of paternalistic housing projects. For instance, Clifton R. Bechtel, Chairman of the City Planning and Zoning Committee of the Chicago Real Estate Board, writes:

> "The Chicago Real Estate Board has based its plan on co-operative individualism. There is nothing new or original about this. It came to Illinois with the first settlers. The individual stood on his own feet and carried on alone as far as he could go, and when the job was too big, he called in the neighbors and finished the job.
>
> "Our Committee feels the place to start is with the people who own the land. They feel paternalistic policies are bad, in that they tend to weaken the citizen. Many proposals are advocated as emergency measures, but even emergency measures should be carefully scrutinized."

Chicago Real Estate Board.—Some of the points of the rehabilitation plan advocated by the Chicago Real Estate Board are:

> "To arouse local public opinion and public spirit so the people themselves—the property owners and residents—can participate in the action."

* * * *

> "To aid in working out the problem along sound economic lines as a commercial and not as a philanthropic proposition."

* * * *

> "To ascertain the kind of people to live in the district; to consider those now living there; to consider those who might be induced to go there. This would mean taking into consideration the earning power of these people in making housing plans that, in cost or rentals, will fit their average income."

Lack of Private Capital.—Attempts have been made in the past to create equity trusts for the purpose of developing low-cost housing areas in which individual property is assembled into large holdings; but there has always been the complication that private capital could not be induced to take the risk because of the many unstable conditions inherent in the proposition. These conditions include instability of low-cost housing areas relative to adjacent areas and uses; the availability of employment for residents; and the ability of residents to pay the rent in such low-cost housing when constructed.

If a region is re-zoned, however, manufacturing is retained and invited to come into the blighted district, and low-cost housing projects are built in appropriate areas relative to manufacturing. There will then be large areas for residential development for housing the higher income groups along the main thoroughfares crossing the district; and it is in these areas that the assembling of property into an equity trust for ownership management and operation, or for any private undertaking, might be conducted with success, because of the stability the district would gain under the new setting.

Limited Profit Agencies.—There is also the possibility of pooling property interests in an equity trust that could obtain funds as a limited-profit housing agency, at interest rates not less than the going Federal rate. The limited-profit housing agency would be required to maintain the low-cost character of the housing, and, therefore, would not come into direct competition with private building enterprise.

The plan of pooling property interests in the formation of a limited profit housing agency would make possible low-cost housing on a much larger scale than any other co-operative limited-dividend corporation, because the Government funds would be available for costs other than that of acquiring the land.

Comments on Questionnaire

Many interesting comments were made on the questionnaire previously referred to, among which the following were most valuable:

(1) Zoning is useful in protecting new sections from blighting influences; but alone it can accomplish little in rehabilitating run-down sections. It cannot be done intelligently unless correlated with a comprehensive city plan.

(2) Re-zoning is needed not so much to protect and increase value as to protect amenity and stability which, in turn, will be reflected in future value. Residential owners want protection of the character of their neighborhoods. Business property owners want reasonable opportunity to use their real estate profitably, consistent with the governing law of supply and demand.

(3) In re-zoning an attempt should be made to meet the following ratios: Highways and streets, 31%; apartments, 15%; homes, 37%; business, 5%; industry, 6%; and recreation, 6 per cent.

(4) Re-zoning should be along scientific lines after a careful survey of the city and of the trends in other cities of similar character; and it should be without property owners' selfish and half-baked notions. The average property owner will not devote enough time to the problem to understand it and support re-zoning in his own interest, especially in areas now over-zoned for business use.

(5) Re-zoning should be planned so that each area which shows definite trends can develop in conformity with that trend, carefully considering the relation to abutting districts of higher or lower classification, and giving special attention to the developments permitted along common boundary lines.

(6) A metropolitan committee is recommended consisting of property owners, realtors, architects, builders, traffic men, financiers, attorneys, physicians, health authorities, officials of street railway and bus companies, planning agencies, and engineers. Planners should profit by former zoning; present their findings to the City Council and have them enacted into a law with "teeth" in it; make re-zoning conditions non-political.

(7) The major objectives of zoning are born of economic necessity. Zoning is an effective stabilizing factor in real estate and is an important contribution

to enhancement of all civic values. It is a needed preventive and a civic insurance against blight and slum conditions.

(8) A practical supply-and-demand analysis (covering both the present and potential need for residential, business, and industrial uses, based upon up-to-date, accurate, detailed information, and upon comprehensive planning considerations) is the proper formula for re-zoning. This will include much more drastic limitations on the size of residential buildings, as well as on the area devoted to business.

(9) The main problem, in addition to reducing business zones, is to reduce vast areas zoned for large apartment houses, but built up with single dwellings, "duplexes," and flats, to zones protecting existing uses. The principal arguments in favor of such re-zoning are: Additional protection to home districts; stabilization of property values and community development; prevention of blight caused by evacuation of home owners to newer protected areas when apartment houses intrude; and prevention of new assessments to provide additional public utilities to replace those overtaxed by the increased population brought in by the intruding apartment houses.

(10) Re-zoning should be based on a land-use survey of the present built-up areas, and upon an estimate of the future trend of growth of undeveloped areas and their suitability for different classes of uses.

CONCLUSIONS

1.—The evidence is quite convincing that cities, particularly the larger cities, must be re-zoned if urban area uses are to be proportioned correctly and located in advance of any further city development.

2.—Zoning plans must be recast in many respects to provide an equitable and sound basis for tax valuation.

3.—Land-use surveys show conclusively that absorption of urban areas occurs in accordance with definite factors, which can be applied with reasonable accuracy in apportioning city areas for various uses.

4.—Property now static because of improper zoning must be cleared of its fictitious use as a prerequisite for its development to its highest and best use.

5.—The method of re-zoning must first be tested against a land-use survey; then an intelligent analysis of the results of the survey must be made to determine present land use and trends; third, the factors of use absorption must be determined, based on this survey, but taking into consideration surveys made in other cities of similar character; and, finally, these factors must be applied intelligently not only to present developed areas, but also in the allocation of use areas for the future absorption of vacant properties in accordance with anticipated growth.

6.—Upon completion of the fact-finding survey and the recasting of use, an official plan should then be prepared by an official planning body.

7.—A strong citizens' council should be formed to serve as an advisory group to the planning body and to throw the weight of all civic organizations back of the re-zoning plan—a body that should be free politically, and frank, firm, and thorough in its job of informing the people in regard to the need for, and benefits of, re-zoning.

8.—This council should co-ordinate its efforts and those of the planning body with the tax assessor's office in the preparation of a plan providing an equitable valuation of property for taxation purposes, developing information as to the ultimate beneficial effects that will result from having property revitalized and put in line for developing into its highest and best use.

9.—It should be the policy of the city to seek approval of re-zoning on the basis of an official zoning plan based upon a scientific analysis of land-use information. Such a plan should be used in the determination of use absorption for the present and future; and planners should develop a direct official method of putting it into operation through the instrumentality of the citizens' council, which should strive in an intelligent and aggressive manner to remove the blight of present fictitious zoning and secure the adoption of a re-zoning plan for equitable taxation.

10.—A vital factor in the re-zoning and rehabilitation of blighted areas is the correction of the conditions that are causing manufacturing and industry to decentralize. It should be the purpose of the citizens' council to re-zone and to re-value blighted district property for taxation in a manner that will retain, as well as attract, additional diversified manufacturing, a most essential requirement if employment is to be found that will make it possible for the residents in these areas to live in low-cost housing.

11.—The citizens' council should adopt a policy that will embrace the paternalistic method of rehabilitating blighted areas, because it will find application in large areas of the lowest income groups. In addition to the plan of Federal loans and grants to municipalities in the present policy of decentralizing the housing administration, consideration should be given to the alternate method of permitting property owners to assemble their property under some form of trustee plan that will permit the development of large areas of low-cost housing with the aid of Government loans and grants. The first method places the ownership and administration of a low-cost housing project perpetually with the governmental body; the second method would make possible the development of much larger areas for low-cost housing, and would place the ownership and administration in the hands of the property owners themselves.

12.—The policy adopted by the citizens' council should also include the non-paternalistic method, because this will enable property owners to pool their interest in some form of trusteeship that can operate as a limited profit housing agency. Both the paternalistic and the non-paternalistic methods of low-cost housing are necessary for the rehabilitation of blighted districts. The non-paternalistic method should include some appropriate plan that will permit the property owners themselves to become a factor in the rehousing process by pooling their interests into some form of trusteeship that will greatly amplify the scope of rehabilitation and obviate the transfer of too much real estate to the governmental authority.

Finally, with the following factors assured: (1) The stabilization of property use in blighted areas; (2) the correction of factors that are now operating to decentralize manufacturing and industry; (3) the development of low-cost housing projects by any of the methods proposed; and (4) the full co-operation of the city in the matter of utilities and the creation of badly needed park areas in locations that will serve as buffers between manufacturing property and residential property—it is a foregone conclusion that private capital will be attracted on a large scale to the construction of housing for the higher income groups. They will be especially attracted along the boulevards and major and minor streets crossing the blighted territory, particularly if proper attention is given to the design and construction of these thoroughfares, a matter that will be feasible in the rehabilitation process.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

WATER SUPPLY ENGINEERING[1]

PROGRESS REPORT OF COMMITTEE OF SANITARY ENGINEERING DIVISION

The Chairman of the Committee of the Sanitary Engineering Division on Water Supply Engineering has had some correspondence with the members of the Committee and with the executives of the Sanitary Engineering Division regarding possible changes in the form of the report. There was some sentiment in the Committee in favor of placing more emphasis on the matter of trends in water supply engineering and omitting notes of water-works construction in progress, floods, and unusual events. As the consensus of opinion seemed to favor the form of report which has been used for several years past, that form has again been adopted for the 1937 report.

Water-Works Construction in Progress

Industrial Supply for Birmingham, Alabama.—This supply was described briefly in the 1936 report.[2] Construction on the large rock-fill dam and the approximately 28 miles of transmission and distributing main from 60 in. to 16 in. in diameter has continued during the year, and the work is expected to be completed during 1938. The total cost is estimated at approximately $6 000 000.

Metropolitan Water District of Southern California, Supply from Colorado River.—Construction continues rapidly on this great project of building the Parker Dam, on the Colorado River, and an aqueduct (392 miles long) across country to the Metropolitan Water District in and around Los Angeles, Calif. It is predicted that the first installation will be ready for use in 1939. Coffer-dams up stream and down stream at the Parker Dam site were completed and much of the concrete in the dam has been placed. The foundation extends 235 ft below the stream bed and the dam projects 85 ft above the stream bed, making its total height 320 ft. It is of the arch type. The aqueduct will have a capacity of 1 605 cu ft per sec when all the parallel pipes in the inverted siphons are installed, but only a part of these lines and some of the pumps will be included in the initial installation.

Note.—Written discussion of this report will be transmitted directly to the Chairman for the information of the Committee.

[1] Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 20, 1938

[2] *Proceedings*, Am. Soc. C. E., April, 1937, p. 693. (The following corrections are to be noted for the 1936 report: On p. 694, Lines 6 and 8, change "Orange" to read "Owens"; and on p. 697, Line 12, change "1862" to read "1852.")

Owens Valley Supply for Los Angeles, California.—This work which includes a dam 90 ft high and 750 ft long at Grant Lake, and a dam 167 ft high and 550 ft long at Long Valley, as well as a tunnel 11.3 miles long, will be ready for service during 1938.

Denver, Colorado.—The additional reservoir and Moffat Tunnel with various supply conduits for bringing a new high-service supply into Denver will be completed during 1938. The pipe line includes a reinforced concrete pipe 54 in. in diameter for a head of 400 ft.

Central Valley Project, California.—The United States Reclamation Service is making a beginning on the construction of this long-considered plan for bringing the surplus waters of Northern California south to the delta and Suisun Bay regions near San Francisco, and to parts of the San Joaquin Valley which, in recent years, have suffered greatly from insufficient water. This project, the total cost of which is estimated at approximately $170 000 000, includes Kennett Reservoir on the Upper Sacramento River which will have a capacity of at least 3 000 000 acre-ft (1 000 billion gal) behind a masonry dam (recently renamed Shasta Dam), 420 ft high, a smaller regulating reservoir a few miles down stream from the Kennett Reservoir, the Contra Costa Canal to supply water to the Suisun Bay region near San Francisco, a series of pumping plants to supply this water to the Contra Costa Canal and parts of the San Joaquin Valley, and the Friant Reservoir on the San Joaquin River where a reservoir of 450 000 acre-ft (150 billion gal) will be formed behind a dam 250 ft high. The growth and prosperity of California are absolutely dependent on the utilization of the available water supply, since so much of the occupation of the State is concerned with the growth of fruit, nuts, vegetables, and grain on territory which is productive only when irrigated and is then very productive.

The Metropolitan Water District, Massachusetts.—This large project for taking an additional supply from the Ware and Swift Rivers, 50 to 60 miles west of Boston, Mass., and bringing it by tunnel, 25 miles long, to the existing Wachusett Reservoir, which is about 35 miles west of Boston, is now nearing completion. The Quabbin Dam behind which a reservoir of 415 billion gal will be impounded is the last major item in the present program. This is the dam, it will be recalled, which has been previously described as having a core-wall constructed of pneumatic caissons, the maximum depth being about 90 ft. Its construction includes the placing of about 2 500 000 cu yd of hydraulic fill. The total cost of this project was estimated at about $65 000 000.

Augusta, Georgia.—Work has been started on a 15 mgd filter plant for Augusta, Ga. The estimated cost is $500 000 of which $180 000 is to be money supplied by the Public Works Administration (PWA).

General Study of Water Supply in New Jersey.—A new impetus was given to the study of the complicated water supply problem of Metropolitan, and other parts of, New Jersey by the conference at Trenton, N. J., on December 22, 1937, of the League of Municipalities. It is estimated that the combined capacity of the existing supplies of Northeastern New Jersey, assuming that cross-connections were made to permit combined use of all sources, will become inadequate sometime in the period between 1943 and 1947. A more general realization of the need of prompt action has resulted from these recent discussions.

Delaware River Supply for New York, N. Y.—This new supply of 540 mgd was described briefly in the 1936 report,[3] at which time the contract for the first work, including thirteen shafts, had just been opened. Construction of these shafts is well advanced and a contract has been let for exploratory caissons for the dam for the Rondout Reservoir. Bids were received in November, 1937, for the remaining shafts and most of the 40-mile tunnel between West Branch Reservoir and Hillview Reservoir which is the southerly end of the tunnel. The final awards on these contracts have not yet been made, owing to discussions relating to the unusually high wage scale and consequent high bids for the work.

Charleston, South Carolina.—A tunnel 18.6 miles long and 7 ft in diameter was completed in such fast time that it cannot properly be reported under "work in progress," having been completed in about seven months following August 1, 1936, under the direction of James E. Gibson, M. Am. Soc. C. E. This tunnel was designed to bring about 50 mgd of water from the Edisto River, to Charleston, in order to secure the location in that city of a new mill to be built by a paper company. The tunnel was in marl and was constructed from seventeen shafts and two portals. The excavation was done in part by blasting, followed by compressed-air spades, and finally finished with carpenters' adzes. The total cost was about $1 000 000.

Filter Plant for Chicago, Illinois, Water Supply.—Many years of experimentation in the filtering of the waters of Lake Michigan at the fine experimental plant, constructed and operated to learn the best methods of filtering this water, have at last borne fruit in the appropriation of $4 545 000 as the initial appropriation on a $20 000 000 filtration plant for the South Side. Money was also appropriated for additional deep-rock tunnels on the South Side.

FLOODS AND OTHER ACCIDENTS TO WATER SUPPLIES

Ohio River Flood.—The Ohio River floods of January, 1937, surpassed all others recorded during the 175-yr. occupancy of the Valley by white men, in the reach from Wheeling, W. Va., to Cairo, Ill. In that reach 1 500 000 people in more than 100 communities were without water service for periods of from several days to several weeks. Exceptional floods are recorded in February, 1884, and in March and April, 1913, these floods having a flow estimated at Louisville, Ky., as about three-quarters that of the 1937 flood and about 12 ft lower in stage. An old record describes a flood of 1773, which may have been higher than the 1937 flood, but occurring in an unrestricted river failed to reach the height of the 1937 flood by about 4 ft. A report by Captain S. J. Horn, Board of Engineers, stated that the 1937 flood could have been higher, as snow was conspicuously absent when it occurred; the run-off at Louisville in the 1937 flood was approximately 8 in. as compared with about 6 in. in the highest flood of 1884.

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[3] *Proceedings*, Am. Soc. C. E., April, 1937, p. 694.

This flood brought the realization to Ohio River cities, which might well be shared by many cities on other rivers, that their relatively short background of experience fails to record maximum flood conditions and that a greater factor of safety should be provided in water-works design and construction. Louisville and Evansville, Ind., are already proceeding with measures which will insure water-works service under flood conditions several feet above those of 1937. Cincinnati has had an investigation made. In general, it is much easier to protect the works against interruptions of service than it is to store water over the period when the plant would otherwise be down. In most cities the cost of insuring the continuity of supply during floods is not greater than the cost of storing water to provide for an interruption of even a day or two.

Failure of Marshall Creek Dam, in Kansas.—On September 20, 1937, a dam constructed by the Works Progress Administration (WPA) to form Wyandotte County Lake near Kansas City, Kans., failed when nearing completion. The dam was 1 550 ft long; it had a maximum planned height of 84 ft, and was completed within 4 ft of the top. The slopes were generous, being 1 on 2 near the top, flattened to 1 on 3 at the bottom, most of the water side being 1 on 3. The middle and up-stream faces were constructed of impervious material placed in 8-in. layers and rolled with sheepsfoot rollers. The down-stream part of the dam was constructed of loose rock and earth rolled in 18-in. layers. The impervious parts were controlled as to moisture content and degree of compactness in the modern manner now common for earth dams.

The failure included about 700 ft in length of the central part of the dam and resulted in a nearly vertical face along the center of the dam 40 to 50 ft high, down stream from which the material flowed in the manner which has been noted in connection with the failure of some hydraulic-fill dams and in the failure of the Lafayette Dam, of the East Bay Municipal District of California. A very rough surface resulted and the toe of the dam moved out about 60 ft.

Accounts state that an investigation by a board of engineers was to be made, but the report is not available at present. The foundation under the dam was known to be somewhat soft, and a foundation failure similar to that at the Lafayette Dam has been assumed to be the cause in the absence of detailed information. The Committee notes that means of controlling the compactness of the artificial fill of earth dams appears to be more advanced than methods of testing the foundation regarding its ability to support such fill.

Failure of Fruit-Growers Reservoir Dam near Austin, Colorado.—This dam, 36 ft high and 910 ft long, storing about 33 000 acre-ft of water, failed due to saturation, on June 15, 1937. A slip a few years ago, also due to saturation, was corrected by installing a drainage system, and subsequent to that another slip occurred producing conditions so dangerous that the owners cut a ditch 4 ft wide at a place where the structure was only 5 ft high, in order to lower the water; the flow of the water quickly enlarged the channel so that the entire contents of the reservoir were discharged in 9 hr, causing damage estimated at $300 000. The original cost of the dam was $22 000.

Epidemic at Springfield, Missouri.—An epidemic of dysentery involving about 25 000 cases, and a following one involving about 240 cases of typhoid fever, both in July, 1936, in Springfield, Mo., caused a corresponding legal epidemic which included seventy-seven suits against the Water Company, in the first of which the jury awarded about $3 000 damages. These epidemics have stimulated much study as to cause.

Increase in Water Demands Caused by Air Conditioning

For some years past it has been apparent that estimates of future increase in water consumption must include a suitable allowance for air conditioning. Where water is reasonably cool and cheap a lavish use will result with the cooling water running to waste. The number of air-conditioning installations has increased approximately 1 400% since 1933 and slightly less than one-half of them depend on public water supplies for their cooling and conditioning.[4] The Chicago Loop District is cited as an example of the heavy draft from air conditioning which has overtaxed sewers and has threatened to overtax even the generous capacity of the water system, so that cooling towers are being required for future installations.

On Long Island, New York, scores of wells have been sunk for air-conditioning purposes under requirements that the cooling devices be of non-corrodible metal, water-tight, and the water be returned to the ground through other wells. Heating of the ground has occurred in some of these cases. The technique is new and much is still to be learned as to these return installations. Similar problems have arisen in the Plain States of the country which have no summers.

Special Committee of the Society on Hydraulic Research

This Committee has five projects under way: (1) Traveling waves on steep slopes; (2) curves in open channels; (3) phenomena of intersecting streams; (4) the conversion of kinetic into potential energy; and (5) sedimentation at the confluence of rivers.

Miscellaneous Matters of Jurisdiction over Water Sources

Who Should Control Stream Pollution?—The 1937 regular session of Congress had before it the Barkley-Lonergan Bill which proposed to endow Federal officials with the authority to require the abatement of pollution through Court procedure. A great deal of opposition to this provision has been voiced, it being urged by many sanitary engineers and others active in this field that the Federal officials should aid by investigating stream-pollution conditions and advise as to action to be taken, but that the actual pollution control should be left in the hands of the States and their political sub-divisions. The Vinson Bill embodies this opinion. Persons of this opinion offer the method of State pacts as a means of doing such work, where two or more States are involved.

Federal Authority to Control Drainage Basins.—The Norris Bill, which was also before the regular 1937 Congress, provided for the setting up of seven drainage basin conservation authorities covering the entire United States. Under the terms of this bill, the approval of the authority would have to be obtained before any project could be undertaken, that would have a bearing on the stream or water supply condition in any area. This bill also involved the question of State rights and concentration of power in the hands of Federal officials, and no final action was taken on it. The New England States have united in opposition to such Federal control in connection with two pacts submitted by them to Congress for flood control in the Merrimack and the Connecticut Rivers.

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[4] *Engineering News-Record*, October 7, 1937.

Pennsylvania Sets Up Charge for Taking Surface Water.—The 1937 Pennsylvania Legislature set up the machinery for the review of all the water appropriations that had been made within the State and made provision that the State Water Authority could impose a charge of not more than $1 per 1 000 000 gal as one of the conditions under which future allocations of surface water within the State might be made. This is a new trend in the control of water resources by a State.

Water Investigation in Oregon.—An exhaustive study of the water resources of the State of Oregon is being made by a committee consisting of eleven men from Federal departments and bureaus with some consulting engineers and representatives from State Departments. The study will extend over several years and its cost is estimated as about $1 000 000, including about $250 000 for the installation of additional stations for observing precipitation, stream flow, and ground-water conditions.

Trends in Design and Construction of Dams

Methods of Exploration.—Core-drilled holes large enough to be entered by the engineer and the geologist in a study of foundations, were used some years ago at the Pretty Boy Dam, of the Baltimore, Md., water supply. More recently they have been used effectively at Norris Dam, Tygart River and Bluestone Dams in West Virginia, Fort Peck Dam, in Montana, Possum Kingdom Dam, in Texas, and Conchas Dam, in New Mexico.

Submarine acoustic methods were used in discovering caverns and solution chambers at the Dix River Dam in Kentucky. Geo-physical methods, involving simultaneous records of several instruments of the seismographic type, have been found useful for preliminary examination of tentative dam sites. This method is used in explorations for oil and gas.

Use of Models.—Scale models are being used increasingly to check paper designs and computations of dams and channels. A recent case was that of the Parker Dam, on the Colorado River, for the Metropolitan Water District of Los Angeles and vicinity. Models were used to determine the type of dam most suitable for the operating conditions, slope of crest, location of power plant, and other details.

Earth Dams.—The practice is becoming more general of treating the earth dam as a highly technical problem both in the care exercised in the pre-examination of available materials as a guide to choice of type of dam (that is, whether rolled, hydraulic fill, or semi-hydraulic fill), in the drainage, and in the control of the mixture of available material, degree of moisture, and degree of compaction. As was noted under the heading, "Failure of Marshall Creek Dam, in Kansas," methods of testing the stability of earth foundations for dams seem less well advanced.

Rock-Fill Dams.—The methods of minimizing settlement of the rock fills in San Gabriel Dam No. 2, in California (for example, by washing sandy material into the rock fill), was noted in the 1936 report.[5] More careful laying of the stones in the rock fill in the part at and near the up-stream face to serve as a foundation for the water-tight deck has often been practiced. Recently, Algeria has furnished an example of four rock-fill dams, respectively 76, 146, 180, and 233 ft in height, in which rock fill was placed by hand—a method probably economically possible only where labor is cheap; and yet, if repairs are never required, this is a method perhaps not to be dismissed, even in the United States, without consideration.

Masonry Dams.—Gravity dams, arch dams of the constant angle type or other inclined multiple-arch dams, and beam and slab dams all continue to be selected, according to foundation and abutment conditions met, as types to be designed and built. The Committee again notes that the oldest of these types, the gravity dam, still seems to have the most uncertainty as to the forces acting on it and the design required to give a proper factor of safety.

Progress in Manufacture of Portland Cement

Cement for Board of Water Supply, New York City.—The Committee has received from Thaddeus Merriman, M. Am. Soc. C. E., Consulting Engineer to the Board of Water Supply, New York City, information on what is hoped will prove an outstanding advance in the control of the manufacture of Portland cement, contributing to the increased durability of concrete and mortar structures for water supply and other purposes. These improved methods are prescribed in the new specification of the Board of Water Supply of the City of New York for the Delaware Aqueduct. The purpose of these specifications is to secure Portland cement of such uniformity and quality that all concrete made with it will be at least equal to the best 50% which is now being obtained under current conditions of cement manufacture and concrete placing. To date, this purpose has been accomplished.

The new specification prescribes tests and limits for the Portland cement to be used on that project, which look toward securing, in the completed concrete, a binder which will be strong, dense, and resistant to the solvent effects of water and the salts it carries in solution. This binder consists of the products formed by the cement as it combines with the mixing water and, when it has the foregoing qualities, the concrete continues permanent and durable. As long ago as 1914, Mr. R. J. Colony[6] called this desirable binding material the "amorphous constituent." In thin sections under the microscope and in polarized light this constituent shows no structure and is dark in all positions as the stage is rotated. In concretes that show no signs of disintegration this constituent is always prominently developed; on the other hand, in those that are breaking down, the amorphous constituent is present in limited quantity and is often entirely missing.

The research of the Board of Water Supply, begun in 1911, has been directed toward securing Portland cement of such quality that it will uniformly produce a maximum of this desirable constituent. The foregoing specification requires the cement to conform to the usual limitations as to strength and setting time, etc. In addition, upper limits are placed on the iron, the alumina, the gypsum, the magnesia and the alkalis, and lower limits are set for both iron and silica. More importantly, it is required that the cement shall be completely and carefully manufactured. In order to insure this end the specification prescribes the burning temperature and requires each kiln to be equipped with an automatic recording pyrometer. Still further, the "flue-dust" may not be returned to the kilns whenever it carries excessive quantities of the alkalis.

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[5] *Proceedings*, Am. Soc. C. E., April, 1937, p. 698.

[6] "Petrographic Study of Portland Cement," by R. J. Colony, *School of Mines Quarterly*, Columbia Univ., Vol. 36, November, 1914.

The clinker produced by the kilns is a hydraulic material and sensitive to water. Existing standard specifications are silent as to how carefully it shall be handled before being ground into cement. In order to prevent the clinker from deteriorating prior to grinding, it is required, therefore, that only a minimum of water may be used for cooling purposes while the clinker is hotter than dull red and that, after cooling, it shall be stored under cover and protected from the weather. It is also required that the clinker shall be ground into cement within six weeks of its burning. Finally, it is specified that the clinker shall be ground dry; that is, no water may be added in the final grinding process.

The specification also prescribes a test for the causticity of the cement solution for the purpose of minimizing the accident hazard known as "cement burn" to which workers in concrete are subject. A further test limits the "free alkali content" so as to control "efflorescence" and, at the same time, insure conditions favorable to a maximum development of the amorphous constituent.

A new test which discloses the completeness with which a cement has been manufactured is also included. This test is based on the fact that a water solution of sucrose (cane sugar) has the capacity of dissolving large quantities of lime. When cement is put into this solution the uncombined lime which is present will dissolve, together with such other lime as is released during the 2-hr period of the test. A thoroughly burned cement gives up very little lime whereas one that has been under-burned often yields thirty times as much. This test gives promise of almost completely disclosing the quality of a cement. It is easily made—six samples can readily be reported on by one man within 4 hr. As a means of controlling plant operation, it is also of great value.

Under this specification and up to this time about 200 000 bbl of cement have been manufactured by six different mills. All of it has been of uniform quality and has made concrete of high excellence in which the amorphous constituent is largely developed. The concrete shows no signs of shrinkage cracks or crazing, efflorescence is almost entirely absent, and cement burns have been reduced. During setting unduly high temperatures are not developed, and there is no formation of laitance even in lifts as high as 20 ft. Strengths are ample at all ages.

Low-Solubility Portland Cement.—Another development in cement for which great merit is claimed in connection with cement lining for hot-water boilers and hot-water pipes, as well as for pipes carrying cold water, is that of low-solubility Portland cement. This is a highly technical product, costing about four times as much as ordinary cement; but cost of materials is perhaps not so great a factor in the lining of pipes as to make this cost necessarily prohibitive if freedom from caustic water and from deterioration of cement lining is insured thereby.

FLOW OF SILT-LADEN WATER THROUGH RESERVOIRS

In 1935, while water was temporarily being discharged from Lake Mead through the tunnels at the bottom of the reservoir, turbid water was shown three times, although the water in the lake was apparently clear. At the time these turbid discharges occurred, the reservoir was from 70 to 90 miles long and contained between 4 000 000 and 5 000 000 acre-ft. It was found that the silt-laden layer occupied the original river channel and was approximately 2 ft deep, with a sharp division line between the clear water and the silt-laden water.

Similar experiences had been recorded at Elephant Butte Reservoir, where the data available indicated that from four to five days would be required for silt-laden water to pass through that reservoir which was from 30 to 35 miles in length, representing an average velocity of 0.5 ft per sec.

The silt-laden water has generally a higher specific gravity than the water that is encountered in the reservoir. The specific gravity is affected also by the dissolved mineral content and to a lesser degree by the temperature. It is evident that turbid heavy water may travel for 100 miles or more through a reservoir without mixing with the remainder of the water in the reservoir, due to the differences in the specific gravity between the clear and turbid waters.

The experience at Lake Mead raised the question as to whether it was not advisable to provide discharge gates at low levels so that as much as practicable of the silt-laden waters may be removed without necessitating the deposit of the silt in the bottom of the reservoir. As many reservoirs on silt-bearing rivers like Lake Mead have a useful life limited by filling with silt, this study may be of great importance.

DEVELOPMENTS IN CENTRIFUGAL PUMPS

In all probability the most outstanding developments in centrifugal pump design for the past year have been in the submersible deep-well turbine pump and high-pressure boiler feed-pumps.

Submersible Deep-Well Pumps.—In the case of the submersible pump, standard deep-well turbine pump elements are directly connected to a motor designed to operate submerged in water. Three American manufacturers are offering such units. One company offers a pump of European design that encases the motor windings in a stainless steel shield with hydraulic, grease-packed ball-bearings. Water is permitted to enter the motor housing for cooling purposes. Due to the wide gap between the armature and the field, and the friction between the armature and surrounding water, the motor efficiencies are considerably lower than standard conventional motors of the same horse-power.

A second manufacturer encloses the complete motor in a cylinder filled with a di-electric oil, preventing the entrance of water by means of a compression-grease-packed stuffing-box. A third company offers a unit somewhat similar to the second in design, but uses a rotating mercury seal to prevent water from entering the motor-enclosing case. A volatile oil is used as a di-electric which, under a slight heat, forms a gas at the motor armature gap. A patented hydraulic balancing tube equalizes the pressures in the motor casing.

In all three designs remarkably high horse-powers are attained in small diameter frames, due to excellent cooling, as the units are immersed in water. Where large horse-power requirements are necessary motors can be assembled in tandem.

For deep setting and in crooked borings submersible units offer many advantages over the conventional shaft-line pumps. Their use for water supply is increasing and as a matter of interest it may be mentioned that their use for pumping inflammable liquids is increasing.

Boiler-Feed Pumps.—During the past year the trend to higher boiler pressures has necessitated the designing of high-pressure boiler-feed pumps, and centrifugal units are being much used for this service. Boiler pressures requiring 1 800-lb discharge pressures at the pumps are not uncommon, and during the past year (1937) pumps to operate against 2 400-lb pressure have been built. To withstand these high pressures double-case pumps are designed.

Fundamentally, these units consist of an inner horizontally split volute case, in which the rotating element revolves, held in a forged steel outer barrel. The inner volute case is under compression as the full discharge pressure of the pump surrounds the volute case. The outer case, which is low carbon forged steel, carries the full discharge pressure under tension. The rotating element is both statically and dynamically balanced before assembling in the inner case. Impellers are single-suction, mounted back-to-back to balance the hydraulic thrust. In this manner no heavy thrust bearing or balancing drum with its uncertain functioning is required. Stuffing-boxes are water-cooled with bleed-off connections.

Units have been built for capacities up to 1 600 gpm. For small capacities up to 250 gpm similar units have been built to operate in a vertical position. These units are slightly different in construction, as they have but one stuffing-box placed on the suction side of the pump. Efficiencies of more than 70% have been attained with the smaller capacity units, whereas on the larger capacities more than 75% has been attained. Special materials are selected for reliability of operation and for maintaining high efficiency over long periods of operation with little maintenance costs.

Progress in the Design and Construction of Pipe Lines

Concrete Pipes for Higher Pressures.—At Denver reinforced concrete pipe has been used in the 54-in. size for heads as great as 400 ft. There would appear to be no reason, with the perfection which joint construction has now attained, why the use of this material should be limited as to heads any more than the steel upon which its strength depends.

Welded Steel Pipe.—Processes of manufacturing welded steel pipe continue to be developed and improved. In a comparatively few years welding has become the standard and nearly the only method in use.

Cement-Asbestos Pipe.—There has been a considerable growth in the use of cement-asbestos pipe for transmission and distribution uses.

Cast-Iron Pipe.—Cast-iron pipes made centrifugally in sand moulds and in water-cooled metal moulds, as well as pipes cast horizontally in green sand, continue to increase in percentage used as compared with the older type of pipes cast vertically in pits. In sizes of 20 in. and less, the newer types have, for a considerable time, nearly displaced pit-cast pipes. Centrifugally cast pipes are being made in increasingly large sizes and are now available at least in sizes as great as 30 in. and 36 in., as illustrated by the use of a large quantity of these sizes during the past year at Miami Beach, Fla. This happened to be for sewer purposes, but would have been equally applicable to water.

Bolted gland joints with rubber gaskets are having an increased use, particularly for suction and submarine lines. Manufacturers of cast-iron pipe are continually developing improvements in these joints. One company manufactures a ball-and-socket joint with a rubber packer at the equator of the joint to give water-tightness, which flexible joints with lead are likely to lack unless the lead is caulked by injection of lead plugs as in the joint approved by the New York Board of Water Supply. It is reported that the use of this rubber joint has been successful.

Section Committee A-21 (American Water Works Association) on specifications for cast-iron pipe and special castings, has in the hands of the Committee for voting a new specification for pit-cast pipe and a new schedule of thicknesses computed by the method developed by the Committee through an extended series of tests which takes account of both water pressure and trench load. Similar specifications and thickness schedules for the newer types of pipe will be completed soon.

Interior Protection for Pipe Lines.—For some years the attention of pipe users has been continuously directed by many fine committee reports and discussions to the greatly reduced capacity generally found in iron or steel pipes owing to incrustation or tuberculation even in water often regarded as non-corrosive. Spun-cement lining and spun-tar enamel lining are successful answers to this problem as far as new work is concerned, but inquiry indicates that their use advances haltingly in the United States as a whole. The reports from several large manufacturers of cast-iron pipe are shown in Table 1. Company No. 1 estimates that 50% of its lined pipe is sold east of the Allegheny

TABLE 1.—PERCENTAGE OF TOTAL CAST-IRON PIPE OUTPUT THAT IS LINED WITH CEMENT MORTAR

Company No.	Location	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937
1	East of the Allegheny Mountains	....	....	....	....	....	3.6	....	....	....	....	12.9
2	New England	....	....	....	....	....	....	....	....	....	....	....
3	South	....	....	....	....	....	....	7.3	3.6	1.5	0.85	2.6
4	South	....	....	....	....	....	....	....	....	....	6	7
5	South	10	10	10	7	7	4	4	4	4	4	5
6	New Jersey	....	....	....	....	....	....	18	12	18	20	20

Mountains and that 5% of all cast-iron pipes sold are cement lined. Company No. 2 reports little, if any, increase in cement-lined pipes at its Pennsylvania plant, but a steady increase at its New England plant where the percentage is now about 50. Statistics are available only for the Southern plant of Company No. 3, Table 1. Its Ohio plant produces no cement-lined pipe.

The Committee is of the opinion that a study of flow coefficients would indicate justification for a greatly increased use of improved linings, as even with the mildest of waters coefficients of larger pipes are generally found to be less than 100, and coefficients of smaller pipes from 50 to 75.

Effect of the Treatment of Water on Flow Coefficients.—Treatment of water to produce optimum condition as regards freedom from red water and corrosion is steadily being extended, but there is a dearth of facts as to the effects. Red water troubles are generally successfully corrected, but flow coefficients remain in doubt. Reliable tests from several plants, which have come to the Committee's attention, indicate possibly some benefit to flow coefficients from treatment, but far from satisfactory results.

Removal of dissolved oxygen and other gases by vacuum and auxiliary chemical treatment has proved to be very efficient in preserving almost undiminished the flow coefficient of a force main at the Grande Ecaille development of the Freeport Sulphur Company, located in the salt marshes of the Mississippi Delta, approximately fifty miles below New Orleans, La.[7] The water after this de-gasification is not exposed to the air during its passage from the treatment works to the end of the main.

Commercial Methods of Lining Pipes in Place.—Note was made in a previous report of the successful demonstration of a commercial method for placing a cement lining on pipes 36 in. in diameter and larger, the mortar being thrown on by centrifugal force and smoothed by revolving trowel blades.[8] The first job done with this equipment—that is, the placing of a cement mortar lining generally $\frac{1}{4}$ in. thick, in about 27 500 ft of 40-yr old, 48-in. riveted steel pipe in the Newark, N. J., transmission system from the Pequannock Water-Shed, has been described.[9] This work increased the Williams-Hazen coefficient from about 70 to 124, which is about fifteen points higher than the coefficient of this lap-jointed, riveted pipe when new. Recently, a new 48-in. steel main about 3.5 miles long, belonging to the City of Toledo, Ohio, was lined in place with this effective process. The lining was $\frac{3}{16}$ in. thick.

Another process for lining pipe with cement mortar in place (namely, by mandrel pulled through the pipe line against a charge of mortar previously injected into the pipe) has been noted in previous reports.[10] The inventor reports that about 1 600 miles of pipe 12 in., or less, in diameter have now been lined. This includes work in Australia, New Zealand, England, India, South Africa, and the Argentine Republic. Experimentally, an 18-in. pipe has been lined, and the company is prepared to line 20-in. pipe. A representative has recently come to the United States for the purpose of introducing the process.

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[7] *Chemical and Metallurgical Engineering*, April, 1936.

[8] *Proceedings*, Am. Soc. C. E., December, 1934, p. 1478.

[9] *Water Works Engineering*, August 5, 19, 1936.

[10] *Proceedings*, Am. Soc. C. E., April, 1935, pp. 4–11.

Exterior Protection of Pipe Lines.—The Committee reported at some length in the 1936 report on cathodic protection from corrosion by earth and on soil surveys to determine the location of corrosive ground. To recite the details of the continued study and progress on this matter would require a separate report. Much of this work centers in the American Petroleum Institute, the American Gas Association, and the National Bureau of Standards group originally aided by these organizations. This is because long and costly oil and gas pipe lines are frequently located in parts of the country where corrosion from soil is most active.

In the eastern part of the United States the problem is generally so unimportant that cast-iron gas mains have seldom been protected and water pipes have been treated only as an incidental step in the dipping process designed mainly for interior protection. On the Pacific Coast there is a definite trend toward protection of all important pipe lines from electrolytic action. As an indication of the mental attitude, one qualified speaker at a recent conference made the statement that all external pipe corrosion is electrolytic, being caused either by stray currents or by currents set up between the pipe and the soil.

In at least the three largest water-works organizations on the Coast, likelihood of corrosion is determined previous to installation of important lines by point-to-point electrolytic surveys, first over the surface of the ground and, when the trench is open, at various depths along the side of the cut. Protection of the pipe is then designed on the basis of these findings.

Pipe protection may take the form of cathodic protection insuring a negative potential in the pipe with respect to the surrounding soil by the use of motor generator sets, rectifiers, etc., taking power from the local power distributing system or from special power plants. In other cases, insulating coatings are considered sufficient. Where the trouble is due to stray currents, tie lines are installed, draining the current back to the power-house in a harmless manner. For certain oil or gas lines cathodic protection has been accomplished by the use of buried pieces of zinc with suitable wire connections.

Laboratories are maintained in several cities in California with electrical engineers in charge, developing suitable methods and mapping the territory according to the severity of electrolytic conditions.

The conclusion has been reached that marine-laid-down deposits offer the worst conditions, with adobes and clays leading. A well-drained sandy soil gives the least trouble. Conditions in parts of California are particularly bad, without a doubt. Some instances have occurred where cast-iron pipe has become completely destroyed by soil action within a period of less than ten years.

Improved Check-Valves.—Balanced or non-slam check-valves, with improved waterways cutting down loss of head very greatly and thus saving energy costs, are being increasingly used. Valve manufacturers are very active in their development in this field and have utilized hydraulic testing as an aid in this development.

Water-Hammer in Pipe Lines.—This subject is receiving increasing attention. The "Second Symposium on Water Hammer," under the auspices of the American Society of Mechanical Engineers and with the co-operation of the Society, the American Water Works Association, and others,[11] was presented on December 8 and 9, 1937. Papers affecting the water supply art were given on Air Chambers, Resurge Phase of Water Hammer, Penstock Failures, Emergency Closing Valves in Pipe Lines, Water Hammer in Branch Pipe Lines, Water Hammer in Pump Discharge Lines, Water Hammer Tests on Transite Pipe, Water Hammer Problems in Pumping Plants, etc.

The papers are in considerable part mathematical, and simple treatments outlining results applicable to ordinary water supply systems have not yet emerged. A more general understanding is being gained by water supply men of the dangers in force mains owing to sudden power failures, and in gravity lines owing to valve closures in times which are too short compared with the length of time required for a wave of water-hammer to traverse the pipe line and return to its source. A member of the Committee finds that practically all major pipe lines being constructed at the present time are protected by automatic valves, usually of the cone or plug type, so located, designed, and equipped (in some cases with relief opening), as to minimize surge and isolate breaks.

Progress in Knowledge and Utilization of Ground-Water Supplies

The past few years have witnessed continued additions to the knowledge of the limitations of ground-water supplies and of methods of increased utilization. The State of New Mexico passed a law for the conservation of artesian water in 1905, and this law has been improved from time to time so that design, materials, and construction of wells are closely supervised to avoid waste of water. Recently, a State force has been engaged in plugging abandoned wells in Pecos Valley, which were bleeding the artesian formation.

The rich ground-water resources of Long Island have been the subject of much detailed study by State, county, municipal, and private interests, and reliable information obtained and published on the serious over-drafts which have occurred in the western part of the Island within the limits of New York City. Destruction of many wells by intrusion of salt water has resulted. The State, which has jurisdiction over the construction of new wells, has refused permission to sink additional wells in this overworked territory, except where the water is to be used in closed non-corrodible cooling systems and returned to the ground through other wells.

In New Jersey, some similar studies have been made, and some salt-water intrusions discovered; the Attorney General of the State has ruled that the State has control over draft of ground-water. In other States, also, surveys have been made and limitations in use found to be desirable.

Ground-Water Replenishment.—In California, where water is of the highest value because its presence or absence in adequate quantities means success or failure of the fruit and other agricultural interests, which make the wealth of the State, there has been a maximum study and practice in improving ground-water conditions. The Santa Clara Valley Conservation District has been described in this connection.[12] This is the first season of operation of replenishment works and results have been very gratifying. Since the beginning of the dry season about 39 000 acre-ft have been released from the reservoirs, a quantity about equal to the estimated normal pumping over-draft. Ground-water levels in the main central area at the end of September, 1937, were only about 3 ft lower than the high level for the year which occurred in April, 1937. Levels in this section are 8 to 10 ft higher than the 1936 low. The May high of 1937 was about 28 ft higher than the all-time low of 1934 in the central portion of the Valley, 60 ft higher in the upper 12%, comprising the percolation mounds or gravel cones, and 14 ft higher in the lower 26%, comprising the area formerly artesian. Reservoir releases will continue until the rains start, apparently insuring a considerable permanent rise in the water plane. The project is a notable example of utilization of flood waters formerly lost by rapid discharge into the ocean at times when the ground was saturated.

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[11] *Proceedings*, Am. Soc. C. E., April, 1937, p. 700.

[12] *Western Construction News*, May, 1936; and *Engineering News-Record*, July 2, 1936.

PROGRESS IN TREATMENT OF WATER

Treatment Before Filtration.—Developments in the treatment and filtration of water within the past year have not been great. There is a more general realization of the importance of properly conditioning water for filtration and consequent interest in the means of improving this pre-conditioning. The silica naturally present in water has been found to give aid to coagulation with aluminum sulfate, and it also has been found that the addition of silica in a certain form may be used to give still greater aid to the coagulation. The development of silica application has not progressed to the state where predictions may be made as to the extent to which the material will be used in water treatment, but it seems probable that a number of filtration plants will find it helpful enough to justify the cost of the treatment.

Titanium sulfate also has been found to give aid to coagulation, and there may be places where this material can be used to advantage.

Fluorides in water continue to receive consideration, and have been found in many more water supplies than has been previously supposed. The prospects are good that practical means of removal soon will be available. The addition of magnesium sulfate to water, which is later converted to a magnesium hydroxide precipitate, causes some removal of *fluorides.* Tri-calcium phosphate also has been found effective in the removal of this substance. In Arizona, a bone filter has been found effective at a cost not altogether prohibitive.

Removal of Tastes and Odors.—No outstanding developments have been made during the year on the removal of tastes and odors from water. Progress continues in the use of activated carbon for this purpose. The manufacturers of activated carbon continue to improve the adsorptiveness of their material. Objectionable tastes and odors now may be removed from the water of nearly all public supplies at a cost for carbon well within the sum the water-works may afford to expend.

A new method of combining the use of powdered carbon, which is a comparatively cheap material, and the more expensive granular carbon appears to be promising. The dosage of powdered carbon is proportioned to remove tastes and odors, but the water is finally passed through granular carbon which eliminates any residual tastes and odors, but does not lose in effectiveness in the absence of matters causing such tastes and odors.

Some difference of opinion has developed as to the best means of evaluating activated carbon and more study will have to be given to carbon testing.

Mixing Basins and Settling Basins.—There were no developments of importance in mixing and settling basins in 1937. Two-story settling basins have been constructed in the new filtration plant at Milwaukee, Wis., which is now nearing completion. There has been some extension of the use of mechanical flocculators. Mechanical mixing devices are being constructed in nearly all new filtration plants of any size, indicating their superiority over baffled basins.

Cleaning Filters.—Interest is being manifested in keeping filter beds in good condition. The volume of mud balls in filter beds is being measured carefully in all the filtration plants in one State by employees of the State Department of Health. Use of the surface wash as an aid to keeping filter beds in good condition is being extended.

Anthracite Coal.—The use of anthracite coal as the filtering medium continues to increase, although the number of filter beds so equipped is not extensive.

Filter Bottoms.—There are several trial installations of a commercial porous-plate filter bottom. None of these filter bottoms has been in service long enough to form an opinion as to the probable use in the future of this type of bottom.

Application of Copper Sulfate in Reservoirs.—The Bureau of Water-Works of Los Angeles has developed and used with apparent success a method of applying copper sulfate to reservoirs by spreading or blowing it as a powder over the surface of the water in the reservoir.

New Basic Inquiry into Effectiveness of Purification and Sterilization Processes.—During 1937 interest in the efficiency of water purification processes and the adequacy of present standards has received considerable stimulus. The Purification Division of the American Water Works Association has organized the Committee for the Study of Chlorine-Ammonia Treatment to investigate the efficiency of chlorination procedures as used in water-works practice. The rôle of water supply in obscure epidemics of gastro-enteritis was the subject of a panel discussion at the meeting of the Association at Buffalo, N. Y., in June, 1937, the proceedings of which have been recorded.[13] On account of the lack of definite information on these outbreaks, the subject has been assigned for study to the newly formed Coordinating Committee of the Purification Division on Methods of Water Treatment and Laboratory Control. This Committee is also attempting to co-ordinate information on established methods of water purification and the adequacy of existing laboratory methods and purification standards. Very recently a Sub-Committee of the Committee of the American Public Health Association on Research and Standards has been appointed to conduct independent research on water-borne gastro-enteritis and to co-operate with the Coordinating Committee of the American Water Works Association.

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[13] *Journal*, Am. Water Works Assoc., August, 1937.

In addition to the investigations by the foregoing committees, research work on this subject is going on in school and other laboratories, some of which were referred to in the aforementioned panel discussion. The work of Professor Paul J. Beard, at Stanford University, on the longevity of the typhoid bacillus under certain conditions is an example of this important work, all looking toward greater safeguards against water-borne diseases.

Miscellaneous New Designs and Ideas in Water Supply

Rubber for Water-Tight Joints.—In the past, rubber, compressed into joints in concrete, has been used in a few cases in place of bituminous or other inelastic caulking material, to make water-tight joints. A recent development which appears to be logical and deserves trial is the use of a latex rubber joint filler which is not compressed but remains adherent to the sides of the joint and contracts and expands without cracking.

Tanks.—Welding of oil and gasoline tanks resting on the ground has been common practice for some years, and this practice was followed slowly for water stand-pipes, beginning with the welding of thin bottoms resting on the earth and a sand cushion like those of gasoline tanks, instead of on masonry foundations as was formerly water supply practice. Prior to this year (1937) welding of elevated steel tanks was in general limited to supports and riser pipes. So rapid has been the development in welding that at the present time practically all tanks of a capacity in excess of 500 000 gal are welded throughout. This development has taken place within one year as far as elevated steel tanks are concerned.

The City of Sacramento, Calif., has recently completed two 3 000 000-gal elevated tanks of a novel type. They are 144 ft in diameter, 26 ft deep, and have their tops 100 ft above ground. The tanks have walls of welded steel plates, but the floors and supporting structures are of reinforced concrete. The roofs are of redwood.

Dry Ice for Redeveloping Wells.—Successful use of dry ice for redeveloping wells at Orland and San José, Calif., has been recorded.[14]

F. A. Barbour, L. R. Howson,
J. R. Baylis, J. A. Wade,
W. W. Brush, T. H. Wiggin, *Chairman,*

Committee of the Sanitary Engineering Division on
Water Supply Engineering

January 20, 1938.

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[14] *Water Works Engineering,* July 7, 1937.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

MOMENTS IN FLAT SLABS OF VARIOUS TYPES[1]

PROGRESS REPORT OF SUB-COMMITTEE (F) OF THE COMMITTEE OF THE STRUCTURAL DIVISION ON MASONRY AND REINFORCED CONCRETE

INTRODUCTION

Present code requirements for flat slabs, such as the "Building Regulations for Reinforced Concrete," adopted by the American Concrete Institute (A. C. I.) in 1936, apply only to " * * * a series of rectangular slabs of approximately uniform size arranged in three or more rows of panels in each direction * * * " supported on columns having capitals not smaller in diameter, c, than 0.225 times the span length, l. This report presents moment coefficients for flat slabs not covered by these conditions, and includes: (*a*) Panels arranged in two adjacent rows ($c = 0.225\,l$); (*b*) four adjacent corner panels ($c = 0.225\,l$); (*c*) slabs without dropped panels arranged in three or more rows ($c = 0.15\,l$); (*d*) slabs without dropped panels arranged in two adjacent rows ($c = 0.15\,l$); and, (*e*) slabs without dropped panels arranged in three rows consisting of two wide outer rows separated by a narrow "corridor span" ($c = 0.15\,l$).

In the development of design coefficients extensive use has been made of data in "Analysis of Homogeneous Elastic Plates," by H. M. Westergaard, M. Am. Soc. C. E., which was Part II of the paper, "Moments and Stresses in Slabs," by Professor Westergaard and the late W. A. Slater, M. Am. Soc. C. E.[2] Based on the work of Mr. N. J. Nielsen (who assumed point supports instead of supports on column capitals, exterior panels simply supported on rigid lintels and slabs without dropped panels), Professor Westergaard deduced moment coefficients for support on column capitals. He presented coefficients for the principal design sections in square interior panels, square exterior panels in two adjacent rows of wall panels, square exterior panels in four adjacent corner panels, and oblong interior panels. The Committee has interpreted and modified these data in accordance with current flat-slab design practice. Its deductions are based upon the mathematically determined coefficients for square interior panels which are compared with the accepted design coefficients, as exemplified by the A. C. I. regulations for the special cases considered.

NOTE.—Written comments are invited for immediate publication; to ensure publication the last discussion should be submitted **July 15, 1938.**

[1] Presented at the meeting of the Structural Division, New York, N. Y., January 20, 1938.

[2] *Proceedings*, Am. Concrete Inst., Vol. 17, 1921.

The report is confined to slabs with the two-way system of reinforcement and with marginal beams or bearing walls.

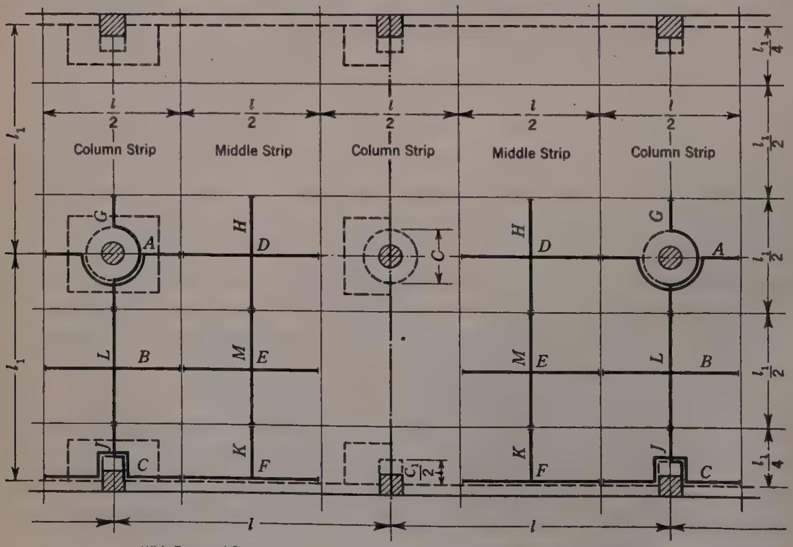
(*a*) PANELS ARRANGED IN TWO ADJACENT ROWS ($c = 0.225\,l$)

Professor Westergaard gives moment coefficients for "two adjacent rows of wall panels with lintel beams," the panels being square and the load distributed uniformly over all panels.[3] Coefficients from these data are included in Table 1 for the principal design sections, *A*, *D*, *B*, and *E*, parallel to the marginal beam, and in Table 2 for the principal design sections, *G*, *L*, *H*, and *M*, perpendicular to the marginal beams. The position of the design sections is shown in Fig. 1.

TABLE 1.—MOMENT COEFFICIENTS FOR DESIGN SECTIONS PARALLEL TO MARGINAL BEAMS; TWO ADJACENT ROWS OF SQUARE PANELS

Item No.	Description	NEGATIVE MOMENTS			POSITIVE MOMENTS		
		Coefficients for Section*:					
		A	*D*	*A + D*	*B*	*E*	*B + E*
1	Original data†	−0.62	−0.22	−0.84	+0.29	+0.26	+0.55
	Modified to Apply to Slabs:						
2	With dropped panels	−0.64	−0.19	−0.83	+0.28	+0.28	+0.56
3	Without dropped panels	−0.59	−0.21	−0.80	+0.30	+0.30	+0.60

* See Fig. 1. † *Proceedings*, Am. Concrete Inst., Vol. 17, 1921.



With Dropped Panels Without Dropped Panels

FIG. 1.—TABLES OF MOMENTS IN PERCENTAGE OF M_0 OR M_0' (EQUATIONS (1) AND (2)). FLAT SLABS ARRANGED IN TWO ADJACENT ROWS OF WALL PANELS, WITH TWO-WAY REINFORCEMENT, WITH AND WITHOUT DROPPED PANELS, AND WITH FLARED COLUMN CAPITALS

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[3] *Proceedings*, Am. Concrete Inst., Vol. 17, 1921, see the third of the five groups in Table VI.

Item No. 1, Tables 1 and 2, may be modified to apply to slabs with dropped panels (Item No. 2) and without dropped panels (Item No. 3). The modification is in accordance with similar adjustments made when coefficients were established in the A. C. I. Regulations and consists in multiplying Item No. 1, Tables 1 and 2, by the factors underlined in Table 3. For illustration, the coefficients for the negative moment in a column strip, when there are three or more rows of panels, are 0.48 (Item No. 1, Table 3), 0.50 (A. C. I. Regulations "with dropped panel"), and 0.46 (A. C. I. Regulations "without dropped panel"), which gives the correction factors: $\frac{0.50}{0.48} = 1.04$, and $\frac{0.46}{0.48} = 0.96$, by use of which the modified coefficients in Table 1 may be computed: $-0.62 \times 1.04 = -0.64$ and $-0.62 \times 0.96 = -0.59$.

TABLE 2.—MOMENT COEFFICIENTS FOR DESIGN SECTIONS PERPENDICULAR TO MARGINAL BEAMS: TWO ADJACENT ROWS OF SQUARE PANELS

Item No.	Description	NEGATIVE MOMENTS			POSITIVE MOMENTS		
		Coefficients for Sections*:					
		G	*L*	*J*	*H*	*M*	*K*
1	Original data†	−0.51	−0.11	....	+0.22	+0.09	....
	Modified to Apply to Slabs:						
2	With dropped panels	−0.53	−0.10	−0.13	+0.21	+0.10	+0.05
3	Without dropped panels	−0.49	−0.10	−0.12	+0.23	+0.10	+0.06

* See Fig. 1. † *Proceedings*, Am. Concrete Inst., Vol. 17, 1921.

TABLE 3.—ADJUSTMENT FACTORS APPLIED TO WESTERGAARD'S MOMENT COEFFICIENTS; THREE OR MORE ROWS OF SQUARE PANELS

Item No.	Description	NEGATIVE MOMENTS		POSITIVE MOMENTS	
		Coefficients for Strips*			
		Column	Middle	Column	Middle
1	Original data†	−0.48	−0.17	+0.21	+0.14
	Modified to Apply to Slabs:				
2	With dropped panels	$\frac{0.50}{0.48} = \underline{1.04}$	$\frac{0.15}{0.17} = \underline{0.88}$	$\frac{0.20}{0.21} = \underline{0.95}$	$\frac{0.15}{0.14} = \underline{1.07}$
3	Without dropped panels	$\frac{0.46}{0.48} = \underline{0.96}$	$\frac{0.16}{0.17} = \underline{0.94}$	$\frac{0.22}{0.21} = \underline{1.05}$	$\frac{0.16}{0.14} = \underline{1.14}$

* See Fig. 1. † *Proceedings*, Am. Concrete Inst., Vol. 17, 1921.

For this case and for other similar subsequent cases, the following moments[4] are used for the design sections at a discontinuous edge (see Fig. 1 for position of design sections):

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[4] Taken in accordance with A. C. I. Regulations, see Tables 3 and 4, Chapter 10.

Design Sections	Moment
J	$0.25\ M_G$
K	$0.25\ M_H$
C	$0.90\ M_A$
F	$0.667\ M_D$

in which M_G, M_H, etc., are moments for Sections G, H, etc. The moment coefficients, determined as briefly described, are summarized in Table 4 for square panels, $\frac{l}{l_1} = 1.00$.

TABLE 4.—MOMENT COEFFICIENTS FOR TWO ADJACENT ROWS OF SQUARE AND RECTANGULAR PANELS ($c = 0.225\ l$)

WITH DROPPED PANELS; COEFFICIENTS FOR THE FOLLOWING RATIOS OF $\frac{l}{l_1}$:			Strip	Design section (see Fig. 1)	WITHOUT DROPPED PANELS; COEFFICIENTS FOR THE FOLLOWING RATIOS OF $\frac{l}{l_1}$:		
0.75	1.00	1.33			0.75	1.00	1.35
(*a*) PERCENTAGES OF M_0' (SEE EQUATION (2)):							
−0.61	−0.64	−0.67	Column	*A*	−0.57	−0.59	−0.62
+0.26	+0.28	+0.32		*B*	+0.28	+0.30	+0.34
−0.56	−0.58	−0.61		*C*	−0.51	−0.53	−0.56
−0.21	−0.19	−0.16	Middle	*D*	−0.24	−0.21	−0.18
+0.31	+0.28	+0.23		*E*	+0.33	+0.30	+0.24
−0.13	−0.12	−0.10		*F*	−0.15	−0.13	−0.11
(*b*) PERCENTAGES OF M_0 (SEE EQUATION (1)):							
−0.56	−0.53	−0.51	Column	*G*	−0.51	−0.49	−0.47
+0.24	+0.21	+0.20		*H*	+0.26	+0.23	+0.21
−0.14	−0.13	−0.13	One-half Column	*J*	−0.13	−0.12	−0.12
+0.06	+0.05	+0.05		*K*	+0.07	+0.06	+0.05
−0.09	−0.10	−0.11	Middle	*L*	−0.09	−0.10	−0.11
+0.08	+0.10	+0.11		*M*	+0.08	+0.10	+0.11

It has been customary to apply coefficients for square panels to rectangular panels also, provided the ratio of long to short side of panel does not exceed 1.33; but it is often desirable to adjust the coefficients for the effect of unequal spans. The adjustment has been made in Table 4 by multiplying the values established for $\frac{l}{l_1} = 1.00$ by the factors underlined in Table 5.

To illustrate the use of Table 5: When there are three or more panels, coefficients for the negative moments in a column strip are given by Professor Westergaard[5] as $0.40 + 0.08\frac{l}{l_1}$, which equals 0.480 for square panels but 0.460 or 0.507 for rectangular panels, depending upon the relationship of the design section to the longer side of the panel. Consequently, the adjustment factors for unequal panel sides in the case being illustrated become: $\frac{0.460}{0.480} = 0.96$; and, $\frac{0.507}{0.480} = 1.05$, from which (in the case of Section A, Table 4): $-0.64 \times 0.96 = -0.61$; and, $-0.64 \times 1.05 = -0.67$. The values in Table 4 are percentages

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[5] See Fig. 26 in "Analysis of Homogeneous Elastic Plates," Pt. II, "Moments and Stresses in Slabs," by H. M. Westergaard and W. A. Slater, *Proceedings*, Am. Concrete Inst., Vol. 17, 1921.

TABLE 5.—ADJUSTMENT FACTORS FOR RECTANGULAR PANELS APPLIED TO MOMENT COEFFICIENTS FOR SQUARE PANELS ARRANGED IN THREE OR MORE ROWS

Item No.	Design section	NEGATIVE MOMENTS		POSITIVE MOMENTS	
		Coefficients for Sections*			
		Column	Middle	Column	Middle
1	Original data†	−0.048	−0.17	+0.21	+0.14
	Modified to Apply to Slabs:				
2	$\frac{\text{Short span}}{\text{Long span}} = 0.75$	$\frac{0.460}{0.480} = \underline{0.96}$	$\frac{0.190}{0.170} = \underline{1.12}$	$\frac{0.195}{0.210} = \underline{0.93}$	$\frac{0.155}{0.140} = \underline{1.11}$
3	$\frac{\text{Long span}}{\text{Short span}} = 1.33$	$\frac{0.570}{0.480} = \underline{1.05}$	$\frac{0.143}{0.170} = \underline{0.84}$	$\frac{0.237}{0.210} = \underline{1.13}$	$\frac{0.113}{0.140} = \underline{0.81}$

* See Fig. 1. † *Proceedings*, Am. Concrete Inst., Vol. 17, 1921, Fig. 26.

of M_0, or M_0', which, in current design regulations, are expressed algebraically as follows:

For design sections perpendicular to the discontinuous edge,

$$M_0 = 0.09\, Wl \left(1 - \frac{2c}{3l}\right)^2 \dots\dots\dots\dots (1)$$

For design sections parallel to the discontinuous edge,

$$M_0' = 0.09\, Wl_1 \left(1 - \frac{c + c_1}{3 l_1}\right)^2 \dots\dots\dots\dots (2)$$

in which, W is the total load on a panel, and l, l_1, c, and c_1 are defined in Fig. 1. Coefficients different from 0.09 may be justifiable in certain types of flat slab and should possibly be somewhat greater than 0.09 for ribbed construction used in flat-slab layouts. In end panels, the moment coefficients for sections parallel to the end beams are the same as those in Table 4 for corresponding sections parallel to the discontinuous edge.

(b) FOUR ADJACENT CORNER PANELS ($c = 0.225\, l$)

Coefficients for "four adjacent corner panels with lintel beams," given by Professor Westergaard, are listed as Item No. 1, Table 6. These values are modified, using factors in Table 3, to obtain coefficients for slabs with, and without, dropped panels. The moment coefficients in other design sections are determined as was done for sections J, K, C, and F in the case of panels arranged in two adjacent rows ($c = 0.225\, l$), and the coefficients are tabulated for

$\frac{l_2}{l_1} = 1.00$ in Table 7. (Note that sections marked *J*, *K*, *C*, and *F*, in Fig. 1, correspond to sections marked *D*, *E*, *C*, and *J*, in Fig. 2.) Adjustment for rectangular panels $\left(\frac{l_2}{l_1} = 0.75, \text{ or } 1.33\right)$ is made by use of the factors in

TABLE 6.—MOMENT COEFFICIENTS FOR DESIGN SECTIONS FOR FOUR ADJACENT SQUARE CORNER PANELS

Item No.	Description	NEGATIVE MOMENTS			POSITIVE MOMENTS		
		Coefficients for Sections*					
		A	*G*	*D*	*B*	*H*	*E*
1	Original data†	−0.57	−0.10	....	+0.27	+0.17	....
	Modified to Apply to Slabs:						
2	With dropped panels	−0.59	−0.09	−0.15	+0.26	+0.18	+0.07
3	Without dropped panels	−0.55	−0.09	−0.14	+0.28	+0.19	+0.07

* See Fig. 2. † *Proceedings*, Am. Concrete Inst., Vol. 17, 1921.

Table 5. The design sections, *A* to *J*, inclusive, in Table 7, are shown in Fig. 2, together with the definition of l_2, l_1, c, and c_1 which are used for the determination of M_0' in accordance with Equation (2).

TABLE 7.—MOMENT COEFFICIENTS (PERCENTAGES OF M_0') FOR FOUR ADJACENT SQUARE AND RECTANGULAR CORNER PANELS ($c = 0.225\ l$)

WITH DROPPED PANEL; COEFFICIENTS FOR THE FOLLOWING RATIOS OF $\frac{l_2}{l_1}$:			Strip	Design section (see Fig. 2)	WITHOUT DROPPED PANEL; COEFFICIENTS FOR THE FOLLOWING RATIOS OF $\frac{l_2}{l_1}$:		
0.75	1.00	1.33			0.75	1.00	1.33
−0.57	−0.59	−0.62	Column	*A*	−0.53	−0.55	−0.58
+0.24	+0.26	+0.29		*B*	+0.26	+0.28	+0.32
−0.51	−0.53	−0.56		*C*	−0.48	−0.50	−0.52
−0.14	−0.15	−0.16	One-half column	*D*	−0.13	−0.14	−0.15
+0.07	+0.07	+0.08		*E*	+0.07	+0.07	+0.08
−0.13	−0.14	−0.15		*F*	−0.12	−0.13	−0.14
−0.10	−0.09	−0.08	Middle	*G*	−0.10	−0.09	−0.08
+0.20	+0.18	+0.15		*H*	+0.21	+0.19	+0.15
−0.07	−0.06	−0.05		*J*	−0.07	−0.06	−0.05

(*c*) SLABS WITHOUT DROPPED PANELS ARRANGED IN THREE OR MORE ROWS ($c = 0.15\ l$)

Current design regulations give total moments within design sections that are one-half panel wide. These total moments (according to Professor Westergaard[6]) change only slightly with changing values of $\frac{c}{l}$. The negative moment per unit width within the column strip varies considerably with $\frac{c}{l}$

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[6] *Proceedings*, Am. Concrete Inst., Vol. 17, 1921, see summary in Table III.

as Professor Westergaard has illustrated. He divides the negative moment in column strips into "moments across edge of capital" and "moment outside capital,"[6] a distribution which is repeated in Table 8 for $c = 0.15\,l$ and also for $c = 0.225\,l$, the special case in the A. C. I. Regulations. Moments per unit width in terms of $\frac{M_0}{l}$, computed from Item No. 1, Table 8, are plotted in Fig. 3(*a*). For illustration, dividing the total moment across the column capital, $0.318\,M_0$, by the width of the capital, $0.15\,l$, gives the moment of $2.12\,\frac{M_0}{l}$ per unit width.

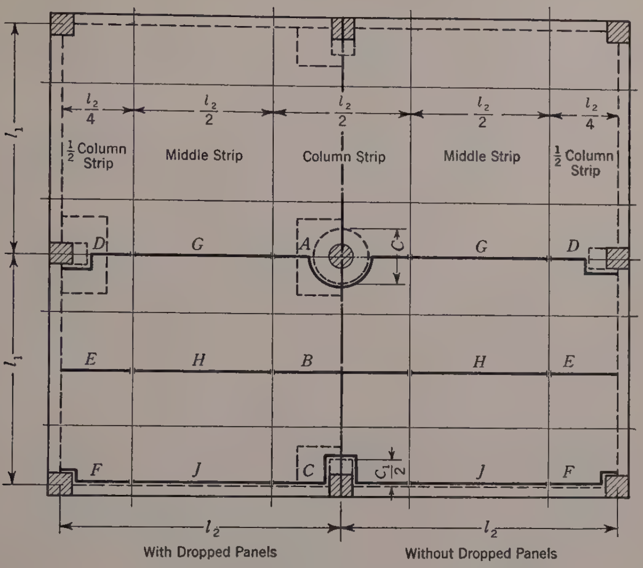


FIG. 2.—TABLE OF MOMENTS IN PERCENTAGE OF M_0' (EQUATION (2)). FLAT SLABS ARRANGED IN FOUR ADJACENT CORNER PANELS, WITH TWO-WAY REINFORCEMENT, WITH AND WITHOUT DROPPED PANELS, AND WITH FLARED COLUMN CAPITALS

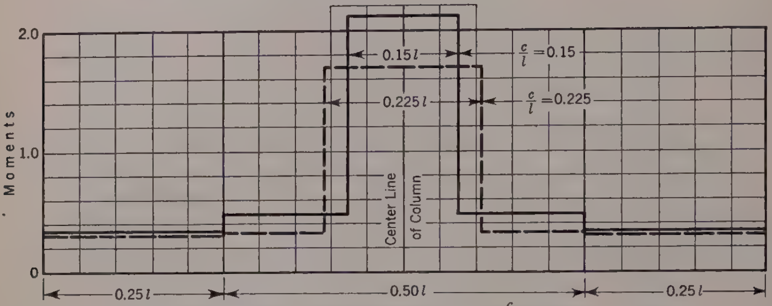
TABLE 8.—MOMENTS PER UNIT WIDTH IN SECTIONS INDICATED; $\left(\frac{c}{l} = 0.150;\ \text{AND}\ \frac{c}{l} = 0.225\right)$

Item No.	DESCRIPTION	ACROSS COLUMN CAPITAL		OUTSIDE COLUMN CAPITAL		MIDDLE STRIP	
	Ratio of $\frac{c}{l}$	0.150	0.225	0.150	0.225	0.150	0.225
1	Original data*	0.318	0.387	0.165	0.096	0.170	0.166
2	Moments per unit width	2.12	1.72	0.47	0.35	0.34	0.33

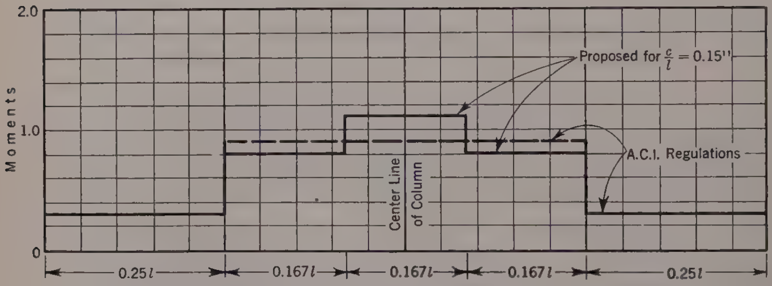
* *Proceedings*, Am. Concrete Inst., Vol. 17, 1921, Table III.

As illustrated in Fig. 3(*a*), moments per unit width increase at the column capital when its diameter decreases, and the ratio of increase is $\frac{2.12}{1.72} = 1.23$ when c decreases from $0.225\,l$ to $0.150\,l$.

Two of the distribution moments are plotted in Fig. 3(*b*), the dotted line corresponding to the A. C. I. Regulations (without dropped panels), and the solid line representing values proposed for $c = 0.15\,l$. For $c = 0.15\,l$, the column strip is divided into three equal strips each having a width of $\frac{0.50}{3} \times l = 0.167\,l$—one "center column strip" and two "outer column strips."



(*a*) THEORETICAL DISTRIBUTION, FOR $\frac{c}{l}$=0.15 AND 0.225



(*b*) DISTRIBUTION OF MOMENTS IN INTERIOR PANELS ACCORDING TO A.C.I. REGULATIONS AND AS PROPOSED FOR $\frac{c}{l}$=0.15 IN.

FIG. 3.—DISTRIBUTION OF MOMENTS IN INTERIOR PANELS

The negative moments are computed accordingly as $1.23 \times \frac{0.46\,M_0}{0.50\,l} \times 0.167\,l = 0.19\,M_0$ in the center column strip (for the value, $0.46\,M_0$, see Item No. 3, Table 3) and $(0.46 - 0.19)\,M_0 = 0.27\,M_0$ in the two outer column strips. The positive moments are $\frac{1}{3} \times 0.22\,M_0 = 0.073\,M_0$, say, $0.08\,M_0$, in the center column strip (for the value, $0.22\,M_0$, see Item No. 3, Table 3), and $(0.22 - 0.08)\,M_0 = 0.14\,M_0$ in the two outer column strips. Since the middle strip need not be divided, both negative and positive moments remain equal to $0.16\,M_0$ (for the value, $0.16\,M_0$, see Item No. 3, Table 3).

Moment coefficients adjusted for a change in $\frac{c}{l}$ are listed under $\frac{l}{l_1} = 1.00$ in Table 9 assuming the slab to be without dropped panels. Adjustments for ratios of panel sides equal to 0.75, or 1.33, are made in Table 9 by multiplying the values for $\frac{l}{l_1} = 1.00$ by the factors established in Table 5. The positions of design sections, *A* to *Q*, in Table 9 are shown in Fig. 4.

TABLE 9.—MOMENT COEFFICIENTS FOR SQUARE AND RECTANGULAR SLABS WITHOUT DROPPED PANELS, ARRANGED IN THREE OR MORE ROWS, $\frac{c}{l} = 0.15$

Strip	Design section (see Fig. 4)	RATIO OF $\frac{l}{l_1}$		
		0.75	1.00	1.33
Center column	*A*	−0.18	−0.19	−0.20
	B	+0.09	+0.10	+0.11
	C	+0.07	+0.08	+0.09
	D	−0.16	−0.17	−0.18
Outer column	*E*	−0.26	−0.27	−0.28
	F	+0.17	+0.18	+0.20
	G	+0.13	+0.14	+0.16
	H	−0.23	−0.24	−0.25
Middle	*J*	−0.18	−0.16	−0.14
	K	+0.22	+0.20	+0.16
	L	+0.18	+0.16	+0.13
	M	−0.12	−0.11	−0.10
Marginal	*N*	−0.11	−0.12	−0.12
	O	+0.07	+0.07	+0.08
	P	+0.05	+0.06	+0.06
	Q	−0.10	−0.10	−0.11

TABLE 10.—MOMENT COEFFICIENTS FOR THREE ROWS CONSISTING OF TWO WIDE OUTER ROWS SEPARATED BY A NARROW "CORRIDOR SPAN"

Strip	Design section (see Fig. 5)	RATIO OF $\frac{l}{l_1}$		
		1.00	0.50	0.00
Center column	*A*	−0.19	−0.21	−0.24
	B	+0.10	+0.10	+0.10
	C	+0.08	+0.08	+0.08
	D	−0.17	−0.19	−0.22
	N	−0.19	−0.19	−0.19
Outer column	*E*	−0.27	−0.31	−0.35
	F	+0.18	+0.19	+0.20
	G	+0.14	+0.14	+0.14
	H	−0.24	−0.28	−0.31
	O	−0.27	−0.27	−0.27
Middle	*J*	−0.16	−0.18	−0.21
	K	+0.20	+0.25	+0.30
	L	+0.16	+0.16	+0.16
	M	−0.11	−0.13	−0.14
	P	−0.16	−0.16	−0.16
Marginal	*Q*	−0.12	−0.13	−0.15
	R	+0.07	+0.07	+0.08
	S	+0.06	+0.06	+0.06
	T	−0.10	−0.12	−0.13
	U	−0.12	−0.12	−0.12

In this report as well as in Professor Westergaard's studies, column capitals have been considered circular. When they are made square, or rectangular (as is desirable in many designs), there is reason to make an adjustment in the ratio of $\frac{c}{l}$ given as 0.15 in the tables for circular capitals. It is proposed to substitute for a round capital with an area, $\frac{\pi}{4} \times c^2$, an equivalent square capital with an area, s^2, the effect of the two capitals being considered identical when $\frac{\pi}{4} \times c^2 = s^2$ or,

$$s = \frac{c}{2}\sqrt{\pi} = 0.89\,c = 0.89 \times 0.15\,l = 0.133\,l \ldots\ldots\ldots\ldots (3)$$

For illustration, a flat slab with panels 15 ft square without dropped panels may be designed for the coefficients in Table 9 if s is not less than $0.133 \times 15 = 2$ ft. With columns 24 in. square, no capital is required. With panels 21 ft square, $s = 0.133 \times 21 = 2.8$ ft; if columns are 18 in. square, the projection of the capital is $\frac{2.8 - 1.5}{2} \times 12 = 8$ in.

(*d*) Slabs Without Dropped Panels Arranged in Two Adjacent Rows ($c = 0.15\,l$)

Moments in flat slabs with two rows of panels, and with $c = 0.15\,l$, may be determined by assuming the same relative distribution of moment between center and outside column strips as was established in connection with "Slabs Without Dropped Panels, Arranged in Three or More Rows." For illustration, with a total negative moment of $0.59\,M_0$ in the column strip (see Section *A*, Table 4), the moment becomes $1.23 \times \dfrac{0.59\,M_0}{0.50\,l} \times 0.167\,l = 0.24\,M_0$ in the center column strip and $(0.59 - 0.24)\,M_0 = 0.35\,M_0$ in the two outer column strips. These and other coefficients based on values from Table 4 are listed for ratios of $\dfrac{l'}{l} = 0.00$ in Table 10, the design sections, *A* to *U*, being as shown in Fig. 5.

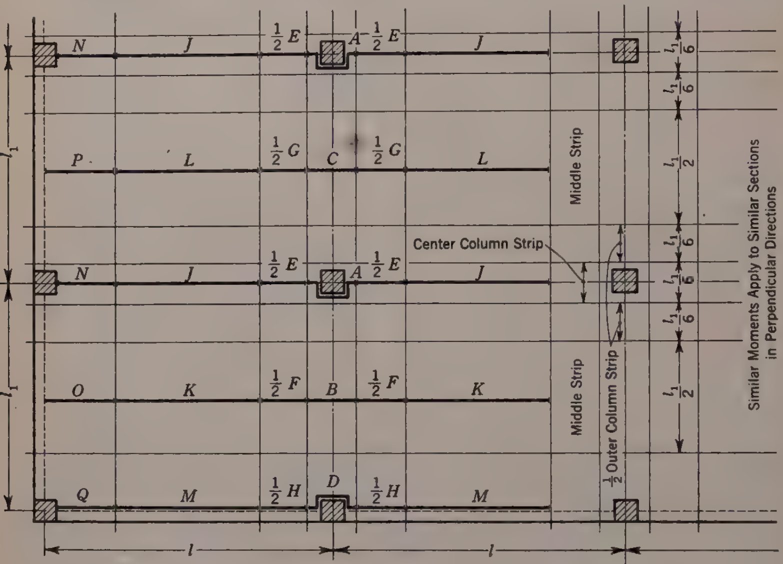


Fig. 4.—Positions of Design Sections, *A* to *Q* (See Table 9)

(*e*) Slabs Without Dropped Panels Arranged in Three Rows Consisting of Two Wide Outer Rows Separated by a Narrow "Corridor Span" ($c = 0.15\,l$)

The framing plan in Fig. 5 has three adjacent rows of panels, the outer panels being square whereas the middle row, the "corridor span," has rectangular panels with a width, l'. If $\dfrac{l'}{l} = 1.00$, the flat slab in Fig. 5 has three equal bays, a case for which coefficients are given for square panels in the center column of Table 9. These coefficients are repeated under a ratio of $\dfrac{l'}{l} = 1.00$ in Table 10.

The coefficients for $\frac{l'}{l} = 0.00$ are for two adjacent rows of panels. For values of $\frac{l'}{l}$ between zero and unity (that is, for a corridor span with varying width) coefficients may be established by interpolation between the values given for the limiting cases. If the intermediate span width is $l' = 0.50 \times l$, the coefficients for design sections marked on Fig. 5 and given in the center column in Table 10 may be taken as the average values of those for $\frac{l'}{l} = 1.00$ and 0.00.

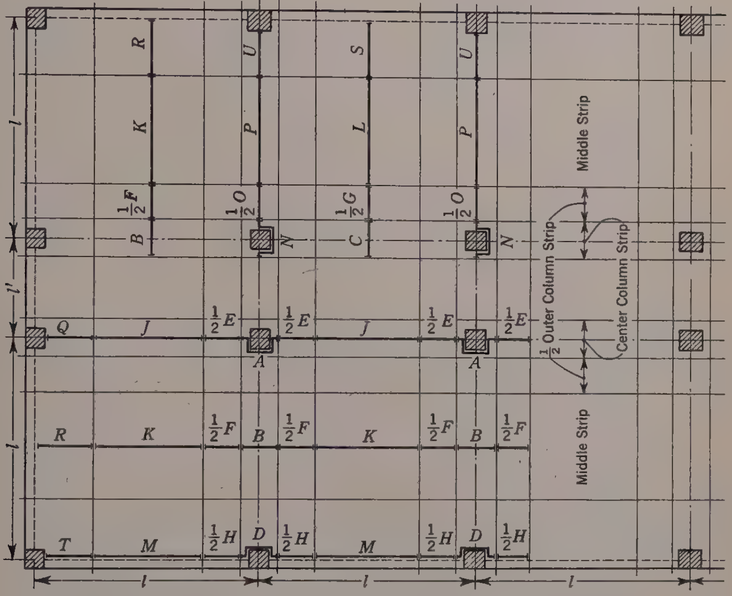


FIG. 5.—POSITIONS OF DESIGN SECTIONS, *A* TO *U* (SEE TABLE 10)

ACKNOWLEDGMENTS

In this report, moment coefficients have been established for flat-slab types not included in the A. C. I. Regulations, by developing data presented by Professor Westergaard and published by the American Concrete Institute.

Respectfully submitted,

LEO H. CORNING,
THORBJÖRN GERMUNDSSON, *Chairman*,
Sub-Committee (F) of the Committee
of the Structural Division on
Masonry and Reinforced Concrete

January 20, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ENGINEERING ECONOMICS AND PUBLIC WORKS
A SYMPOSIUM
Discussion

By J. K. FINCH, M. AM. SOC. C. E.

J. K. FINCH,[60] M. AM. SOC. C. E. (by letter).[60a]—Although few engineers would question the soundness and force of Mr. Fay's arguments for the future planning of public works, it is quite obvious that very great practical difficulties will be encountered in putting them into effect. There are clearly two major phases of this planning movement.

On the one hand, there is the necessity for foresight in planning public works—a need for engineers to encourage comprehensive planning rather than a hand-to-mouth practice in public construction if such works are to be better adapted to future public needs and requirements. In this respect they must recognize that many existing public works agencies are not set up to undertake such comprehensive planning effectively, and it is doubtful whether the fundamental changes in organization, tenure of office, etc., necessary to permit or to encourage the development of a long-range policy, can be effected. Apparently, the practical approach to the problem is not through reforms in existing public works agencies, but through setting up separate planning boards or commissions, which will be independent of changes in political party and will give particular attention to the longer range viewpoint. This movement is growing rapidly. City and regional planning boards are increasing in number. It was reported in 1935, for example, that there were forty-five State planning agencies, and many municipalities have city planning boards. The attack on this problem, therefore, has been through this more or less indirect plan—far from ideal, perhaps, and still, in many cases, lacking in implementation to make the movement fully effective—but apparently the most promising and practical line of attack yet developed.

NOTE.—This Symposium was presented at the meeting of the Engineering-Economics and Finance Division, Boston, Mass., October 7, 1937, and published in February, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the Symposium.

[60] Renwick Prof. of Civ. Eng., and Director, Summer School of Surveying, Columbia Univ., New York, N. Y.

[60a] Received by the Secretary December 24, 1937.

On the other hand, the further proposal to use public works as a means of reducing the fluctuations in the construction industry, while fine in theory, seems discouragingly remote in the possibilities of achievement. It is dependent, of course, on the aforementioned future planning movement but, in addition, unless the United States abandons its democratic form of government, public opinion must, as it always has, continue to play an important part in the initiation of actual construction. There may be little difficulty in speeding up public construction in periods of depression as has been done in recent years, but the idea that the Government can retard public spending when prosperity is at its crest sounds well in theory but impossibly impracticable in practice. It might be hoped that the Government could have in hand some well planned and useful projects which can be undertaken in periods of depression, but, in all probability, the really pressing, and, therefore, most important, works will have been built when times were good and their construction was demanded by the public.

Turning to the papers by Messrs. Mead and Riggs, many readers would accept their statements of fact as to the false business economy, or, too frequently, total lack of such considerations, in the construction of the works described. Many engineers would also condemn in an equally forceful way the apparently deliberate suppression of the engineers' reports on these projects and the later attempt to place the onus for the initiation of these economically useless and imposingly wasteful expenditures on the engineer. Many engineers would also agree with Mr. Mead that the TVA "yardstick" notion is a political humbug, and a child of wishful thinking and bad bookkeeping—that, in fact, the entire recent Federal viewpoint on the power problem is unreasonable, economically destructive, and eminently unjust. At the same time, many engineers would not agree with the conclusion implied, at least, in these papers, to the effect that the Government should have nothing to do with any project involving power production and should build no works that are not fully justified from the standpoint of business economics.

It seems quite obvious, for example, that the Federal Government is already involved in many projects in which the development of power is incidental (is a "contributory benefit," as the Supreme Court puts it) to works which are clearly within the constitutional scope and recognized function of modern government. The time is long since past when Government can limit its activities to the simple functions which constituted its entire scope of duties in Colonial days. The time has passed when the American people will turn over to private interests the remaining public property of the nation for exploitation and development by such interests and for private gain. It seems obvious that this nation has embarked, for better or for worse, on a policy involving the governmental development of its remaining public properties and resources in what it is hoped will be a wider public interest.

It would appear to the writer, therefore, to be futile and impracticable and thus hopelessly reactionary and completely unconstructive, to damn the present entirely and argue for a return of the "good old days." It is all very well to argue that one should not compromise with thieves and murderers, and it is apparently on this ground that the extremists of the New Deal refuse to listen to the power utilities and the power utilities refuse to compromise with the New Deal. To face the facts, however, one must admit that there are two interests both legitimately involved in the power field and, as in the case of capital and labor, these two interests, private and public, must get together, must reach some fair and just basis for co-operation, rather than continue a wasteful and unnecessary battle at the expense of the public. The present Federal viewpoint, if persisted in, will lead to economic suicide. The present public utility attitude often seems planned to alienate rather than woo public support. The engineer is generally considered to be a man of common sense, and he certainly cannot support the extremists on either side. While admitting much which both sides say, he will insist that a compromise is the only practical solution of the problem.

Much can be said, of course, on both sides of this question, and it will probably be necessary, in order to find a sane solution of the problem, to ignore rather than to attempt to reconcile some of the extremists if a compromise is to be effected. It is unquestionably true, for example, that a radical modification of the recent viewpoint of the Federal Administration is the first necessary move toward peace and co-operation. With the Federal Government going ahead with power developments in fields which private companies already serve, with public funds being used to construct municipal plants which put existing plants out of business, it is small wonder that the private utilities are up in arms and that billions in utility constructions are held up. This policy is destructive of economic values; it results in the waste of public funds, depletes the national income, and retards recovery. No engineer whose viewpoint is based on engineering principles rather than on blind bias and emotion can support such a policy.

On the other hand, few thoughtful engineers would attempt to defend the marking up of capitalization and other abuses in some of the power utilities which characterized the pre-depression years. These companies are granted a practical monopoly of an essential service and enjoy a relatively constant and steadily growing market. It seems clear that the lowest possible rates consistent with adequate service should be the goal of their operations. A low interest rate on a fair valuation is all that should be expected. Yet in the halcyon days before the depression the "merchants of debt" argued that capitalization should be marked up to the top figure which income could justify and, through holding companies and similar means, proceeded to burden some utilities with unnecessary and unjustified capital expense. The engineer must condemn with equal warmth this narrow, unsocial, and, in the long run, economically fatal and unconstructive policy. Clearly, if the public utilities take no steps to put their houses in order, and persist in fighting regulation by fair means or foul, the patience of the American people will ultimately be exhausted and the power business, like the water supply business, will be taken over by Government. In the long run, business must be conducted so as to advance the economic welfare of the nation and serve the public interest, if it is to survive.

The writer knows that Messrs. Mead and Riggs would condemn such utility practices and policy, but they apparently feel that this "public utility attitude" reflects the viewpoint of but a small part of the industry, whereas the present policies which have been advanced and encouraged by the New Deal forces are far more fundamental and dangerous to our national life. Although the writer would admit the force of this argument, when advanced by these experienced and careful analysts, it seems clear that the suggestions made in their papers, while directed toward the correction of an existing evil, cannot form the basis for a constructive and progressive general power policy. Co-operation seems essential. After all, it should be the purpose of both Government and business to advance and strengthen the national economic structure, because it is on this structure that our social and national development must rest. Clearly, the basic policy of both Government and business must be based on this principle and must reflect not a narrow, short-sighted opportunism but a reasonable and forward-looking viewpoint. Engineers are so accustomed to planning not for to-day but for to-morrow, of attempting to predict and to plan for future needs, that they assume that this is a common human trait. They evaluate a policy not solely on its immediate advantages but attempt also to picture its future effects. Unfortunately, this is not a common human trait—the world says "take the cash and let the credit go." A little more foresight in both business and political thinking and in the development of business and public works policies is an essential to the continued growth of American institutions. Accepting as a principle the basic idea that mutual understanding and co-operation on a sound economic basis are essential to the future usefulness and sane development of the power business, the writer is confident that engineers are capable of developing, with fairness and impartiality, the necessary details for such a working agreement.

Finally, those engineers who would adopt as a principle the idea that Government should not undertake to build any works except those which can be shown to be economically justified on a business basis, have apparently forgotten or ignored a basic principle of democratic Government. There are many public works which cannot be thus justified and, furthermore, the decision as to whether they should or should not be built is not a simple matter of engineering opinion. It is a question of public opinion. In short, what shall or shall not be done (within the wide limits of constitutional and other law) is determined solely by the state of public opinion at a particular time and place. The result may be the use of public funds for works which are economically justified, or may be believed to be socially desirable, or even for constructions which represent pure waste and extravagance.

If the individual engineer is opposed to all public works which cannot be shown to be economically sound, he is free, as is every other citizen, to express this as a personal view, but it is an impossible principle for the profession as a whole to bring forward. There are certain basic principles, however, on which the Engineering Profession can agree.

Any work, for example, which does not offer opportunities for adequate financial return to the public treasury through direct service charges, through assessment for benefits, or through the creation of additional values which can be taxed, must be classed from the pure economic standpoint, as a luxury. Presumably, such works can be afforded only when the surplus of public funds permits such construction, or when the public is willing to accept such an additional tax burden as their construction entails. This is not a matter of opinion but a statement of fact.

Now, the engineer is an expert in the investment of funds, public or private, in engineering enterprise. It is his duty not only to design such works to meet the service desired with the utmost skill and economy, but also to advise his client, public or private, as to the probable cost and returns from the investment. The Engineering Profession, therefore, is quite within its scope of duty if it insists that the public shall be advised as to the engineering values of any proposed investment of public funds in a public work of an engineering nature. As far as the writer can see, the engineer's duties as a professional man go no further than this. In fact, if he reads these papers correctly, one of the chief objections which the authors offer to recent Government practice rests on the suppression of adverse engineering reports or the assumption that engineering values can be appraised adequately by a layman. The writer, therefore, feels that a great majority of engineers would support the following statements:

(1) That any proposed investment of public funds in any enterprise of an engineering nature should be investigated by (*a*) competent engineers, not directly concerned with, or interested in, the enterprise, who should be asked to review the project as to (*b*) its feasibility and cost, and (*c*) its economic values, reducing these as far as possible to quantitative terms and stating clearly the limits of precision and certainty of the conclusions; and,

(2) That all such reports be given full publicity.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ECONOMIC DIAMETER OF STEEL PENSTOCKS

Discussion

BY M. H. FRESEN, ASSOC. M. AM. SOC. C. E.

M. H. FRESEN,[45] Assoc. M. Am. Soc. C. E. (by letter).[45a]—The discussions which have been presented, contain much valuable information and constructive criticism.

Economic Diameter Using Weisbach Formula for Friction Loss.—Mr. Monroe and Professors Powell and Barrows prefer to use the Weisbach (also sometimes called Darcy or Chezy) formula for head lost in friction in conduits. Since the paper was prepared, friction factors (Table 6) for use in the Weisbach formula have been made available. For this reason Equations (75) and (76) have been derived.

Using the Weisbach formula for friction head loss (Equation (68) quoted by Professor Barrows), the annual cost of the lost power in a penstock of diameter, D, may be expressed similarly to Equation (5) as:

$$E_f = \frac{18.71\, K'\, L_f\, Q^3\, e\, f\, b}{D^5} \qquad (71)$$

in which K' equals the friction loss factor, f, in the Weisbach formula.

The first derivative of Equation (71) with respect to D is:

$$\frac{dE_f}{dD} = -\frac{93.54\, K'\, L_f\, Q^3\, e\, f\, b}{D^6} \qquad (72)$$

In Equation (72) let $93.54\, K'\, L_f\, Q^3 = G'$, which substituted for G in Equation (24) yields,

$$B = \frac{G'\, J}{D^x} + \frac{M\, J}{D^y} \qquad (73)$$

in which M, J, and B have the values given by Equations (23*b*), (23*c*), and

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NOTE.—The paper by the late Charles Voetsch, M. Am. Soc. C. E., and M. H. Fresen, Assoc. M. Am. Soc. C. E., was published in November, 1936, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: March, 1937, by Messrs. R. A. Monroe, William E. Rudolph, and Peter Bier; April, 1937, by Adolpho Santos, Jr., Assoc. M. Am. Soc. C. E.; May, 1937, by Messrs. Joseph D. Lewin, F. Knapp, and Ralph W. Powell; June, 1937, by H. K. Barrows, M. Am. Soc. C. E.; and September, 1937, by Robert W. Angus, Esq.

[45] Associate Engr., U. S. Bureau of Reclamation, Denver, Colo.

[45a] Received by the Secretary February 7, 1938.

(23*d*), respectively. Letting $x = y = 7$, Equation (73) becomes:

$$D = \sqrt[7]{\frac{J\,(G' + M)}{B}} \qquad \text{(74)}$$

approximately, or in terms of the fundamental factors involved, the economic diameter considering the head, H, is:

$$D = \sqrt[7]{\frac{e\,f\,b\,(93.54\,K'\,L_f + 74.83\,K_T)\,Q^3\,s_g\,e_i}{668\,H\,a\,r\,L_p\,(1 + i)}} \qquad \text{(75)}$$

which corresponds to Professor Barrow's Equation (67), except that the notation of the paper is used. For greater accuracy, Equation (75) (with $x = 7$ and $y = 6$) can be solved for D by a trial-and-error method.

Considering the shell thickness, t, the following equation is derived similarly to Equation (75):

$$D = \sqrt[6]{\frac{e\,f\,b\,(93.54\,K'\,L_f + 74.83\,K_T)\,Q^3}{128.3\,t\,a\,r\,L_p\,(1 + i)}} \qquad \text{(76)}$$

approximately. Equation (76) corresponds to Equation (33) of the paper with the same notation, and is similar to one given by Professor Barrows.[41] For greater accuracy, Equation (31) (with G' substituted for G, and $x = 7$ and $y = 6$) can be solved for D by a trial-and-error method.

TABLE 6.—FRICTION FACTOR, K', FOR USE IN WEISBACH FORMULA, EQUATION (68)

Item No. (1)	Reynolds' number, $R_n = \frac{V\,D}{\nu}$ (2)	Full-riveted pipe (3)	Continuous interior pipe (4)
1	10×10^3	0.0392	0.0340
2	50×10^3	0.0314	0.0256
3	100×10^3	0.0285	0.0225
4	500×10^3	0.0228	0.0170
5	$1\,000 \times 10^3$	0.0207	0.0149
6	$5\,000 \times 10^3$	0.0169	0.0115
7	$10\,000 \times 10^3$	0.0154	0.0104
8	$50\,000 \times 10^3$	0.0130	0.0086
9	$100\,000 \times 10^3$	0.0122	0.0082
..	....	....	....

Based on a study reported in 1936,[46] selected average friction factors (as read from a chart) for use in the Weisbach formula, Equation (68), are given in Table 6, Columns (3) and (4), corresponding to the recommended equations:

$$K' = 0.0043 + \frac{0.160}{R_n^{0.165}} \qquad \text{(77)}$$

for full-riveted pipe; and,

$$K' = 0.0043 + \frac{0.254}{R_n^{0.23}} \qquad \text{(78)}$$

for continuous interior pipe. The values in Column (2), Table 6, are based on

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[41] "Water Power Engineering," by H. K. Barrows, 1934 Edition, p. 357.

[46] "Friction Coefficients of Continuous Interior and Riveted Steel Pipes," by M. B. Karelitz and F. M. Watkins. *Technical Memorandum No. 521*, U. S. Bureau of Reclamation, Denver, Colo., May 14, 1936.

Reynolds' number,

$$R_n = \frac{V\,D}{\nu} \qquad (79)$$

in which ν = the kinematic viscosity of the fluid. For water at 15° C (59°F), $\nu = 1.23 \times 10^{-5}$ sq ft per sec.

Nomogram for Solution of Economic Diameter.—The nomogram, Fig. 2, presented by Mr. Lewin for solving Equations (47), (48), or (49) for the economic diameter, D, directly, constitutes a valuable addition to the paper. To use Fig. 2, the maximum discharge, Q, and the head, H, for shell thickness, must be known. Since it is probable that for different projects the load factor, F, the shape of the load curve, and the corresponding value of the loss factor, f, will vary more widely than the other factors in Equations (11) or (27), the writer prefers to use Equation (48), Fig. 2, which includes the loss factor. The scale for Equation (48), Fig. 2, is such that the diameter read from it is that for unity load factor and unity loss factor. For a smaller load factor and loss factor, the diameter may be read from Fig. 2 (scale for Equation (48)), and corrected by the ratios given in Table 1, Columns (1) and (2). Similarly, Fig. 2 and Table 1 (preferably plotted in logarithmic curve form) may be used to estimate the diameters for assumptions other than those on which Equation (48) is based. It should be noted that Fig. 2 cannot be used to determine the diameter, D, when the head, H, and allowable stress, s_g, give a smaller penstock shell thickness than the minimum thickness permitted by the specifications being followed in the design. For such cases, Equation (33) should be used.

Economic Diameters, in Terms of Head, and Shell Thickness.—For exactly corresponding assumptions, the economic diameter of a steel penstock in terms of the head, H, is about 11% less than the economic diameter obtained when the shell thickness, t, corresponding to H from Equation (6) is used. At first thought this may appear to be inconsistent and incorrect. Fig. 6 has been prepared to show that such results are correct, and will be obtained whether the economic diameters are found from diameter-total annual cost curves similar to Fig. 6, or from solving Equations (27) and (33).

Fig. 6 is based on the following assumptions: Q = 1 000 cu ft per sec; $K_s = 0.34$; $e = 0.85$; $f = 0.20$ (corresponding approximately to a load factor, F, of about 50%); b = \$0.003 per kw-hr; s_g = 15 000 lb per sq in.; $e_j = 0.80$; a = \$0.11 per lb; $r = 0.07$; and $i = 0.15$. In this example, the following results are obtained: (*a*) From Fig. 6, Curve E, for a head, H, of 100 ft, the economic diameter is found to be 10.36 ft; (*b*) from Equation (6), the thickness of shell corresponding to a head of 100 ft is t = 0.225 in.; (*c*) from Fig. 6, Curve D, for a thickness, t, of 0.225 in., the economic diameter is found to be 11.66 ft; (*d*) substituting the foregoing assumed values in Equation (27), and solving, D = 10.36 ft for a head of 100 ft, which agrees with Item (*a*); (*e*) similarly, solving Equation (33), D = 11.66 ft for a shell thickness of 0.225 in., which agrees with Item (*c*); (*f*) the ratio of $\frac{\text{Item }(a)}{\text{Item }(c)} = \frac{10.36}{11.66} = 0.889$ (or, in general, the economic diameter of penstock in terms of head is about 11% less than the economic diameter obtained when the corresponding shell thickness is used); and (*g*) the result of Item (*f*) may also be verified algebraically by a study of Equations (6), (27), and (33), and reference to Table 1.

For corresponding assumptions the results obtained, using the equations, are identical with those determined from a detailed analysis. For all ordinary problems the writer believes an appreciable saving of time will result from using the equations of the paper, compared to a detailed analysis.

Miscellaneous.—Mr. Lewin states that the analysis of the paper is "limited to uniform (that is, horizontal) penstocks." He also questions "whether the diameters of penstocks situated immediately down stream from the dam can be found by the analyses suggested in the paper." The writer assumes that Mr. Lewin refers to Equations (11) and (27) for computing D in terms of the head, for a penstock line of constant diameter. Neither of Mr. Lewin's statements applies as long as the following conditions are fulfilled: (*a*) The cost of the penstock per pound is constant irrespective of the variation of the diameters; (*b*) the top section of the penstock does not require a minimum thickness, imposed either for practical reasons or in virtue of danger of collapse;

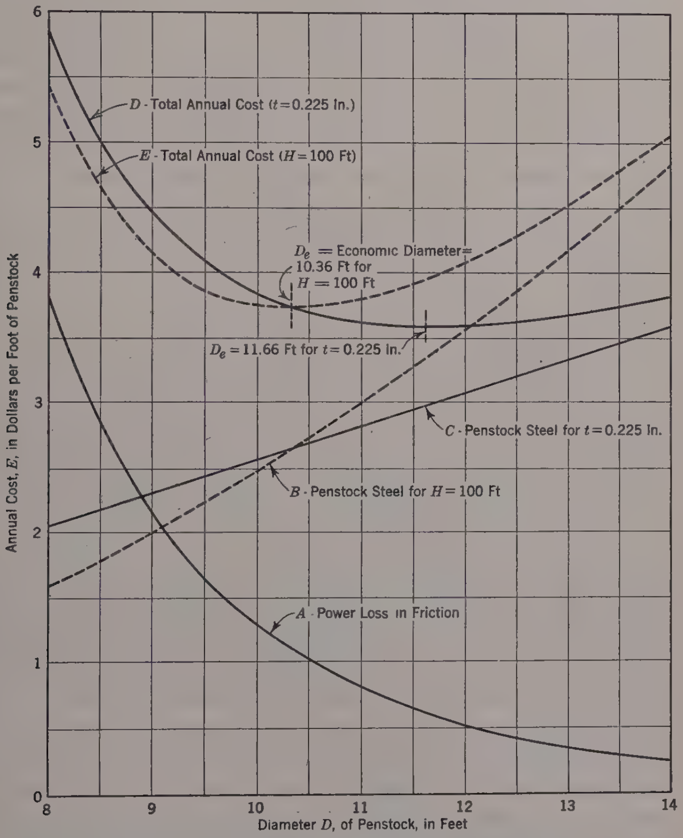

FIG. 6. ECONOMIC DIAMETER OF STEEL PENSTOCK IN TERMS OF HEAD AND PLATE THICKNESS

and (*c*) the average weighted head used in the equations is a true numerical function of the weight of the steel (and, therefore, of the cost) for the penstock, profile, and water-hammer conditions under consideration. Conditions (*a*) and (*b*) are limitations explained by Mr. Knapp.

Equations (51*a*) to (61), inclusive, given by Mr. Lewin, are interesting, but the writer doubts whether the average engineer would have the patience to attempt to apply them. The writer believes it would be more practicable to use Equation (33), applying the method suggested in the paragraph immediately following Equation (33).

Messrs. Santos and Knapp, and Professor Angus discuss water hammer as affecting penstock design. In addition to the Footnote References (30, 44) given by them, several recent papers[47, 48] on water hammer may be of interest to those who wish to pursue this phase of the problem further.

Messrs. Santos and Knapp very properly point out that too much refinement of analytical methods is not warranted, and that all such computations should be used, combined with sound judgment, to arrive at the best solution. The number of significant figures in the numerical constants of the equations in the paper should be reduced to four at the most, probably three being all that are justified.

Mr. Rudolph mentions an interesting application of the formulas of the paper to a problem involving the present value of a saving in the future. This is a phase of the problem in economics which should receive adequate attention.

As mentioned by Messrs. Bier and Lewin, and by Professor Powell it should be emphasized that Equations (11) and (27) apply to high heads where the necessary shell thickness varies directly as the product of the head and diameter; and Equation (33) to the equally important case of low head in which the thickness of the shell is independent of the head, and for small penstocks, independent of the diameter.[49]

Mr. Lewin and Professor Powell comment on the wide range in values of loss factor from Fig. 1 for particular values of load factor. Since no one can predict positively in advance the exact shape and magnitude of a load curve for a projected plant, it is probable that values of loss factor estimated from Fig. 1 are as reliable as those computed by the more accurate method given in the paper, and similar methods mentioned by Mr. Lewin and Professor Powell.

In conclusion, the writer wishes to thank those who have contributed discussions, and hence have added considerably to the value of the paper. The writer appreciates in particular the tribute to Mr. Voetsch[39a] expressed by Professor Powell, and shares in his sorrow that the senior author cannot participate in this closing discussion, and enrich it from the wide experience he possessed.

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[47] Papers relating to water-hammer phenomena in pipe lines, by Lorenzo Allievi, R. W. Angus, F. Knapp, R. T. Knapp, J. N. Le Conte, O. Schnyder, E. B. Strowger, and F. M. Wood, *Transactions*, Am. Soc. M. E., November, 1937, Vol. 59, No. 8.

[48] "Water-Hammer Pressures in Compound and Branched Pipes," by R. W. Angus, *Proceedings*, Am. Soc. C. E., January, 1938, pp. 133–169.

[49] "Penstocks—A Report of the Hydraulic Power Committee," Edison Electric Inst., *Publication No. D15*, December, 1936. This recent report gives much valuable data on design and construction of penstocks. Minimum thickness of steel plates is discussed on p. 5.

[39a] Charles Voetsch, M. Am. Soc. C. E., co-author of this paper, died on February 7, 1935.

Mr. Bier gave an example comparing the economic diameters of a welded-steel discharge pipe for a pumping plant obtained by means of Equation (11), with that obtained by means of a detailed analysis. His data tended to prove that the respective diameters for the two methods were 4.59 ft and 5.75 ft. As shown in Fig. 6, the two methods give the same results, if the same basic data are used throughout.

A check by the writer with Mr. Bier shows that a loss factor, $f = 0.42$, was used in the detailed analysis, whereas in solving Equation (11) a value of $f = 0.13$ was used. The error arose from the fact that Curve E, instead of Curve A, Fig. 1, was used to estimate the loss factor for a load curve of rectangular shape and of constant peak load, with load factor, $F = 0.42$ approximately.

The corrected value of the diameter for a head, H, of 150 ft from Equation (11) for $f = 0.42$ is,

$$D = 4.59 \times \sqrt[6.9]{\frac{0.42}{0.13}} = 4.59 \times 1.185 = 5.44 \text{ ft}$$

for which the corresponding plate thickness from Equation (6) is,

$$t \doteq \frac{0.434 \times 150 \times 5.44 \times 12}{2 \times 13\,500 \times 0.9} = 0.175 \text{ in.}$$

The diameter of 5.44 ft is the correct size for the given head, $H = 150$ ft, unit stress, $s_g = 13\,500$, and joint efficiency, $e_j = 0.90$.

Mr. Bier assumed a plate thickness, $t = 0.25$ in., for the detailed analysis. Using $t = 0.25$ in. and $f = 0.42$, in addition to the values given by Mr. Bier, and substituting in Equation (33), D is found to equal 5.758 ft. Mr. Bier's result of 5.75 ft from the detailed analysis verifies the accuracy of Equation (33). As he states, with $D = 5.75$ ft, $t = 0.25$ in., and $H = 150$ ft, the unit stress, $s_g = 9\,970$ lb per sq in. This shows that the allowable stress, s_g, of 13 500 has not been reached when t is selected arbitrarily at 0.25 in. as the minimum thickness recommended for practical reasons.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

PRACTICAL USE OF HORIZONTAL GEODETIC CONTROL

Discussion

BY ROBERT C. SHELDON, ASSOC. M. AM. SOC. C. E.

ROBERT C. SHELDON,[13] ASSOC. M. AM. SOC. C. E. (by letter).[13a]—Although he was pleased with the several interesting and instructive discussions that were presented, the writer would have been more pleased if the opinions of those engineers interested in small local surveys had been presented.

Several of the discussers have stated that the system of survey computations as presented can be criticized for its lack of accuracy. The system is a combination of geodetic and plane computations and is used in such a manner that the accuracy of the reported position of any point is only limited by the accuracy of the survey methods, or by the accuracy of the basic control system. This is accomplished by the following steps:

(1) Geodetic positions are obtained for scattered points covering the area. This is the base control system, comparable to the control net, both primary and secondary, of the U. S. Coast and Geodetic Survey.

(2) Surveys run between these scattered points are usually computed by plane methods. The computed positions are reported as geodetic positions, the residuals beyond any minute line being carried in feet. This plane calculation gives positions which are not the same as those that would be obtained by a geodetic computation of the same traverse. This discrepancy was called an error in the paper. Table 3(*a*) and (*b*) gives the maximum possible discrepancy. Table 4 shows that this maximum is greatly reduced in practice.

(3) In cases where the points of the base system are so far apart that the error, as shown by Table 3 (the rigid use of which is modified by

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NOTE.—The paper by R. C. Sheldon, Assoc. M. Am. Soc. C. E., was published in April, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1937, by Messrs. Philip Kissam, Ralph Z. Kirkpatrick, E. B. Roberts, H. W. Hemple, J. C. Carpenter, and George D. Whitmore; November, 1937, by R. M. Wilson, M. Am. Soc. C. E.; and January, 1938, by Julius L. Speert, Jun. Am. Soc. C. E.

[13] Asst. Engr., Section of Surveys, The Panama Canal, Balboa, Canal Zone.

[13a] Received by the Secretary February 16, 1938.

Table 4), approaches that of the specified error of the survey, additional triangulation points are put in and geodetic computations are made to obtain their positions. It is seldom necessary, locally, to place any of these additional points.

Mr. Hemple states that Table 3(*a*) and (*b*) is misleading because traverses do not usually extend in a straight line, but consist of a number of broken courses. The discrepancy between positions determined by geodetic computations and those found by plane computations, as well as the convergence of meridians, is caused by the fact that straight lines run on the surface of the earth by transit or other means are curved lines when referred to the sphere as a whole. With the standard assumption that the meridians of longitude all converge at the pole and the parallels of latitude are all parallel to the equator, the effect of this curvature of straight lines varies with their direction, being nothing for lines running due north or due south and a maximum for lines running due east or due west. The degree of this curvature also varies with the latitude, being nothing at the equator and a maximum at the pole. This latter variation would be small over the extent of latitude covered by the type of surveys under discussion, and can be ignored.

To present a clear picture of the relation between convergence of meridians, azimuth discrepancies, geodetic computations, and plane computations, it can be stated that a geodetic computation is the plane computation of a line that has been corrected for convergence of meridians in sections of equal length, those sections being of infinitesimal length.

From the foregoing and from an examination of the formula for computation of geodetic positions given in *Special Publication No. 8* of the Coast and Geodetic Survey,[13b] it is apparent that the discrepancies between plane and geodetic computations as well as the effect of convergence of meridians are almost entirely dependent on the difference in longitude of the ends of the line concerned. It is impossible to find any group of lines, running in any direction, that will have a difference in longitude greater than that of a line of equal length running due west.

Table 3(*a*) and (*b*) gives the maximum difference between positions determined by plane computation and Positions *c* found by geodetic computation, at the various distances from the point of origin. Table 3(*b*) would be entirely too rigid to use as a criterion for spacing points of the primary or secondary system on the Fortieth Parallel of latitude. The writer prepared Table 4 to show that this maximum difference was greatly reduced in practice by balancing the angular difference, due to convergence of meridians, and the error of traverse closure, due to neglect of geodetic corrections. Table 4 shows that for a straight-line traverse of 100 000 ft, the maximum error, from Table 3(*b*), would be 1:500, the unbalanced error of closure would be 1:1 100, and the position error of the worst point would be 1:2 300. Thus, the proportional errors shown by Table 4 were roughly one-half those of Table 3 for unbalanced traverses and one-fourth those of Table 3 for balanced positions. Mr. Hemple has submitted Table 5 showing the results of calculations of three separate traverses. The writer has prepared Table 6 to show the errors that would have been expected in these traverses if Tables 3 and 4 had been used as a guide. It would appear that Table 4 was a conservative estimate of the worst condition ever likely to be encountered in practice.

[13b] See No. (5) of Bibliography of paper, *Proceedings*, Am. Soc. C. E., April, 1937, p. 668.

TABLE 6.—COMPARISON OF EXPECTED TO ACTUAL ERRORS

Traverse No.	Length, in feet	Maximum error expected (Table 3(*b*))	Unbalanced error expected (Table 4(*b*))	Unbalanced error obtained (Table 5)
1	80 700	1 : 750	1 : 1 500	1 : 4 000
2	27 600	1 : 1 500	1 : 3 000	1 : 7 220
3	69 000	1 : 900	1 : 1 800	1 : 11 300

It is unfortunate that Mr. Hemple did not extend Table 5 one place farther and show the discrepancy between the final positions as computed by the geodetic methods and as determined by the writer's method. This final position is the ultimate object of any survey. The writer feels certain that the discrepancy of this final position, in all three cases, would be less than the specified accuracy of the survey.

In the three examples given, even if the plane method of calculation suggested by the writer were satisfactory, he would not advocate its use on traverses of the type shown; that is, on traverses of sufficient importance to justify:

(1) The measurement of horizontal angles over these distances with sufficient accuracy to obtain closures consistently of less than 1′;

(2) Chaining with the accuracy and care necessary to secure 1:10 000 closures;

(3) Observing and computing the temperature corrections necessary; and,

(4) The reduction of the calculations to sea level. The percentage of the total cost of the survey of computing either all or several sections of the traverse by a geodetic computation will be very small.

It is to be clearly understood that the methods presented in the paper are used as an extension of an existing system of primary and secondary control and are not used to obtain that control.

Both Mr. Roberts and Mr. Hemple gave the fact that loop traverses were not permitted in the proposed system as one of its disadvantages. This writer agrees with Professor Kissam that the fact that it does prohibit loop traverses is one of its advantages. Loop traverses have so many of the same qualities of loose ended lines that they are prohibited on any survey that is based on any general control system. On any survey unimportant enough to be based on one tie to the base net, or to the supplementary net, the error taken from Table 3 without any correction for angle balancing would not be large. For example, from Table 3(*b*), a line running for a distance of 10 000 ft from its origin must be at least 20 000 ft long and would be in error at its extremity less than 2 ft—an error of 1 part in 10 000 ft of line and an error of position of 1:5 000.

Mr. Roberts criticizes the linear adjustment of azimuth discrepancies and position closure errors in traverses between geodetic control points. There would be many ways of correcting these azimuth discrepancies and geodetic corrections. The one chosen by the writer was the least accurate but was used because it was far simpler than any other. It requires no extra effort, the geodetic corrections and azimuth corrections being combined with the accidental errors for adjustment. Since these discrepancies and azimuth corrections are kept well below the specified accuracy of the survey the exact balancing of these small corrections is not important.

Traverses that are tied to any base net must originate at one of the points on this net and terminate at, or tie to, another unless they are the objectionable loops previously mentioned. In order to do this they must have a general direction, which is that of the line between the stations, although there may be courses in the traverse that will run exactly opposite this general direction. In this manner all traverses between points on the base system take on generally the characteristics of a straight line between the points. Any adjustment that corrects the closing error has a tendency to bring the positions of the several points nearer the geodetic positions. The fact that, in cases where the individual course runs opposite to the general direction of the traverse, the correction is applied in the opposite sign from the true correction is not as damaging as it would seem, because the corrections applied to the predominant number of courses in the traverse are applied in the right direction and tend to correct the discrepancies of all the positions.

As Mr. Wilson states, the method of transit traverse and computation given in the paper is not new and has been used by many organizations for years. The U. S. Geological Survey uses astronomical observations for azimuth to keep the effect of convergence of meridians at a minimum whereas, in the Canal Zone system, more frequent ties to the base system are used to accomplish the same result.

Professor Kissam asks the question, "Does not the proposed system require reduction from geographic positions to plane co-ordinates, computation by plane co-ordinates, and, finally, reduction back to geographic positions with accompanying loss of accuracy?" The system can best be explained as a plane co-ordinate system in which the spaces between successive minute lines of latitude and longitude are made the planes. The small plane areas that are enclosed by the minute lines are thus made to join at the lines, and there is no distortion of discrepancy when crossing from one to another. The only reduction that is necessary is the one that is necessary on any plane co-ordinate system; that is, the positions of the points on the base geodetic system must be converted to the plane co-ordinate system used. In the proposed system this reduction is made very simple, only the conversion of the seconds of latitude and longitude to feet being necessary.

All points are used with that residual beyond any minute being given in feet.

Many of the discussers have advocated the State-wide plane co-ordinate systems that have been devised by the Coast and Geodetic Survey. These systems have met with much success in the years since the writer began preparing the paper under discussion. Their use is now advocated by many important organizations. They are, as Mr. Hemple points out, the law in at least two States, all of which is part of the reason why the writer feels some hesitancy in presenting the criticism of these State-wide plane co-ordinate systems that should have been made years ago. A greater part of this hesitancy is due to the fact that where they have been adopted and are in use, they will give surveyors and engineers a method of tying their surveys, that will be entirely satisfactory for most uses.

There are some points to these plane co-ordinate systems, however, that should be given full consideration before they are adopted in any State and required by law for land boundary description.

The use of the State-wide plane co-ordinate systems, or any other system that does not use the parallels of latitude or the meridians of longitude as a key, requires the introduction of the X and Y-designation, which is an added factor that engineers, surveyors, but more especially laymen, must learn to understand.

The State-wide systems still have the same objectional feature that is inherent in any plane co-ordinate system, in which the planes are continued for any distance; that is, a grid azimuth must be used. This is certain to cause considerable confusion especially in counties that already have grid azimuths for their own local plane system that will be different from the new State-wide grid. For many years to come, it will be necessary in those places, to mark every azimuth with some identification to show whether it is on the old grid, the new grid, or true azimuth. The Engineering Profession, which is just beginning to understand that there is a true and constant north, is to be now taught, that north is not north, but something else at right angles to the grid system. Although it is a relatively easy problem to identify the system on which any point may be recorded, as the figures will usually be much different, the identification of an unmarked azimuth will be difficult.

One of the advantages of any control system is that any point on a large project is identified and compared to any other point in the system by its co-ordinate positions. In intrastate work the plane system of co-ordinates covering that State does this very well. The tendency now is to forget political boundaries, however, and to base projects and surveys on natural features. Mr. Whitmore mentions that the TVA covers parts of seven States. Seven different State systems of co-ordinates are used and then the geodetic system is used to correlate them all. If a system as advocated by the writer had been adopted, the same amount of field and office work would have placed the entire project on the one system.

The projections of the State-wide plane co-ordinate systems have been arranged so that the curved surface of the earth covering entire States has been squeezed in at the centers and spread out at the edges to form a plane. The degree of distortion caused by this squeezing has been kept below the ratio of 1:10 000. For all surveying work for which it is desired to have an accuracy finer than this, certain corrections must be made. This fact will tend to place a restriction on the more accurate surveys. Surveys that on an unrestricted basis would be run to finer accuracies than 1:10 000 will be run on this standard because of the inclination on the part of engineers not to use any more corrections than necessary. In this manner, progress in more accurate surveying will be retarded.

The use of State-wide plane co-ordinate systems has been mentioned by both Professor Kissam and Mr. Whitmore as a possible approach to the eventual universal use of geodetic control. The writer believes that, rather than being a stepping stone, the use of the State-wide plane co-ordinate systems will act as a wall stopping any future approach to that universal use. Plane surveyors will be content to follow the systems that are given them, and will become, if possible, a lower ranking branch of the profession than they are at present; and the knowledge and practice of geodesy will be confined to the employees of the U. S. Coast and Geodetic Survey.

The basic criticism of the writer to the State-wide systems of plane co-ordinates, or any other system of co-ordinates except geodetic, is that they are unnecessary. When the control system of the U. S. Coast and Geodetic Survey is completed, giving points of which the geodetic co-ordinates are known, at intervals of not more than 25 miles, it will be necessary for some organization, either local, State, or national, to place a number of supplementary points between these basic control points, for the use of local surveyors and engineers. In Pennsylvania, this spacing of supplementary control has been placed at a maximum of 5 miles.[14] These supplementary points will have to be established regardless of the system of computations used. In order to be of unquestioned authority they will have to be located with sufficient accuracy to justify their computation on a geodetic basis. If they are to be placed close enough to be useful to the local surveyor without prohibitive expense they will be close enough so that the maximum error introduced in any line run between them (computed by a plane computation as described in the paper and shown in Table 3(*b*)) will be less than the specified error of the surveys desired. For example, in Pennsylvania, with the maximum spacing of secondary control at 5 miles, the maximum error for the worst case would be the unbalanced, or loop, traverse run for 2.5 miles from the point of origin and returned. From Table 3(*b*) this would give a maximum error of 1:3 500, approximately, which would be the worst case possible and would seldom be encountered. Most lines would be run so that the errors caused by neglect of geodetic corrections would be negligible.

Professor Kissam has revealed the fact that the cost of computing surveys on a truly geodetic basis would be prohibitive for any commercial survey. However, the cost of computations on the system as presented by the writer would be very little. The first and second-order control will have geodetic computations in any case. The spacing of the secondary control will be governed by practical reasons so that it will be used. The cost of running and computing the third order, or traverses for use, will be the same in any case, for the writer's system, for plane co-ordinates, or for uncontrolled surveys.

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[14] "A Standard System of Plane Coordinates for Pennsylvania," by the Pennsylvania State Planning Board, Harrisburg, Pa.

Mr. Carpenter has revealed the fact that the main control net of the U. S. Coast and Geodetic Survey is not nearing completion, but is, in fact, little more than half finished (1938). It would be probably more important at this time to find ways and means of completing this important work and the enormous amount of supplementary control that will be necessary before it can be properly used on any system, than it is to find ways to use it after it is completed. Private engineers and small political units cannot be expected to spend very much money on any type of control other than local. A small part of the money wasted on inaccurately controlled and duplicated surveys would have completed this control system many years ago.

Laws have been passed in two States legalizing the use of State-wide systems of plane co-ordinates for land boundary descriptions. These laws were passed after considerable effort on the part of those interested, and will undoubtedly be legal for a long time. These laws would have been of more permanent value if they had been drawn in a more general manner, so that the primary requirement had been a survey based on geodetic control, leaving the method of making the connection between the geodetic control and the survey to the engineers and surveyors.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

EARTHQUAKE RESISTANCE OF ELEVATED WATER-TANKS

Discussion

By Arthur C. Ruge, Assoc. M. Am. Soc. C. E.

Arthur C. Ruge,[20] Assoc. M. Am. Soc. C. E. (by letter).[20a]—Attention has been called by Captain Heck to the need for further analysis of strong earthquake motions, and he has stated the attitude of the U. S. Coast and Geodetic Survey toward the problem of earthquake-resistant design. Those who are studying the problem recognize the wisdom of the Survey in confining its efforts to collecting data needed for design rather than engaging in the study of design itself, since it thus performs a needful service that could not be done adequately by any existing private institution. Furthermore, the desire to avoid attempting to prove particular theories gives to the work an impersonal character which adds tremendously to its value.

The writer hopes to extend the analysis of earthquake records in co-operation with the Coast and Geodetic Survey along the lines discussed by Captain Heck, and preliminary work in that direction has been in progress for some time. It may be stated herein that the present type of strong-motion accelerometers has been shown to give reliable results and that a convenient means of analysis by integration has been developed.

Mr. Stone notes that the damped spring-element concept can be extended profitably to many other structures. In fact, the patent covering the device does enumerate such uses, and there is every reason to believe that it could be applied satisfactorily in many instances where important structures must be protected against earthquake motions, although the details of the application would be varied considerably to suit individual cases.

The statement of Mr. Stone to the effect that the period of vibration depends upon the initial stresses in the members perhaps needs to be qualified somewhat. It is true that for very small amplitudes the period is

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Note.—The paper by Arthur C. Ruge, Assoc. M. Am. Soc. C. E., was published in May, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1937, by Messrs. N. H. Heck, Mason A. Stone, and H. C. Boardman; and December, 1937, by Franklin P. Ulrich, M. Am. Soc. C. E., and Dean S. Carder, Esq.

[20] Asst. Prof. of Eng. Seismology, Mass. Inst. Tech., Cambridge, Mass.

[20a] Received by the Secretary February 17, 1938.

considerably affected by the initial stresses in the rods. As soon as the amplitude reaches values of some importance (say, more than 0.25 in.) the initial tension has very little effect upon the period—in fact, any earthquake of destructive character will necessarily induce amplitudes large enough to make the dynamical effect of initial tension negligible.

The comment is made by Mr. Stone that the type of tower selected for experiment is a poor one from the standpoint of earthquake resistance, and a broader tower base is suggested. The tower referred to is of a common type and represents what is considered good practice for sprinkler tanks of moderate size and height. Although a wider base might be desirable for optimum efficiency, one cannot, in general, make the base spread very great unless the efficient use of the plant property is of no importance. Of course, the geometrical shape of the panels does not directly enter into the dynamical conditions stated in Conclusion (8), and the designer will normally choose the most economical layout.

As to the absence of a statement of the period and amplitude to be expected in earthquake motions, the omission was intentional. The writer feels that, unless prefaced by an elaborate discussion, any statement of that kind is almost certain to be misinterpreted by the very readers it was intended to help. A glance at the ground motion shown in Fig. 26 will perhaps serve to make this clear.

Mr. Boardman has suggested several interesting designs for tank towers. Such designs are probably best suited to large capacity tanks and, as such, have a very special interest, since for these types the ordinary spring element design would be uneconomical or impracticable. The arrangement shown in Fig. 28 would be well suited for the introduction of damping devices in the diagonals, and a careful design of the columns might permit large deflections. In such a case, the columns might be stiff enough to provide most, if not all, of the spring action and the dampers could be relatively simple friction devices.

The point regarding the use of steeper slopes for the rods to obtain a maximum of elastic tower deflection is well taken, and if the economy is not thereby destroyed, the dynamic design is simplified. Rods of steeper slope mean heavier columns and longer rods to handle; but there may be a gain in economy in some cases. However, the writer wishes to emphasize that some damping would be required in any event unless the structure can safely withstand extremely large deflections, and that there is no cheaper way of improving the dynamical behavior than by the introduction of damping. The presence of the springs in a design merely reduces the necessary damping and makes it convenient to produce the damping action.

The designs shown in Figs. 29 and 30 would doubtless need damping systems that could take the form of simple friction devices actuated by a light diagonal system, the spring action being provided by the columns. A tower such as that in Fig. 27 would have to be designed as a short-period system and no data exist for obtaining the stresses.

Messrs. Ulrich and Carder have contributed a helpful discussion to the paper. The field tests made by the U. S. Coast and Geodetic Survey were most helpful in the interpretation of the laboratory results. It may be stated that the model periods agreed with the prototype periods as closely as could be expected, and that the differences can logically be attributed largely to the degree of initial rod tension in the field.

The question of torsional vibration is properly raised, but the writer is of the opinion that even in conventional towers it would be only a secondary effect, since it would have to be set up principally by whatever lack of symmetry exists in the structure. In a structure equipped with the proper spring elements any torsional vibration would be quickly damped out since the elements come into action from torsion as well as from linear deflection of the tower.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

MEASUREMENT OF DÉBRIS-LADEN STREAM FLOW WITH CRITICAL-DEPTH FLUMES

Discussion

By Edwin S. Fuller, Esq.

Edwin S. Fuller,[13] Esq. (by letter).[13a]—Every one engaged in collecting flow data on streams in which the regimen varies with almost every change in rate of flow, should be grateful that an organization of the character of the California Forest and Range Experiment Station should have concerned itself with the task of attempting to develop a stable rating device for use on such streams. Those who have had much field experience in attempting to stabilize the conditions of flow in flood channels can appreciate the difficulties under which the authors of this paper have been, and are, laboring.

When the rating flume studies were inaugurated in 1935 at the San Dimas Experimental Forest, the writer was privileged to witness some of the earlier tests. His first impression upon seeing the various types of rating flumes that had been constructed, especially those in the testing laboratory down stream from San Dimas Dam, was that these structures would form excellent controlling sections for gages located a short distance up stream from the entrance to the rating flume, and that there might be a splendid chance to compare the ratings of gages so located with those within the rating flumes themselves.

Since readings on staff gages in the approach channels up stream from the rating flumes were included in the comprehensive data gathered during the tests, the publication of these data has been awaited with considerable interest, and great disappointment was felt when it was found that data covering water surface heights in the approach channels were not included. However, the determinations of water surface elevations in the approach channels were later furnished to the writer and will be used in this discussion.

It can be demonstrated that for a stable controlling section of rectangular shape, the rating curve for a gate situated a short distance up stream from

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Note.—The paper by Messrs. H. G. Wilm, John S. Cotton, and H. C. Storey, was published in September, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. R. L. Parshall, and Martin A. Mason.

[13] Hydr. Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

[13a] Received by the Secretary February 7, 1938.

the "control" section will have the form,

$$Q = K\, b\, H^{u} \qquad (11)$$

in which u is an exponent equal to 1.5; Q is the discharge, in cubic feet per second; b is the breadth of the rectangular "control" section, in feet; H is the height of the water surface, in feet, above the lip of the controlling section; and K is a constant depending only on longitudinal channel conditions. Such a rating curve, of course, can be defined from a single determination, at any stage of the discharge.

Applying the water surface elevations in the approach channel to the discharges listed in Table 2, rating curves for the 1-ft, 2-ft, and 3-ft rectangular, San Dimas type, rating flumes, were constructed on rectangular co-ordinates. The relation between the ordinates and the subtangents for these curves shows that in each case the points can be fitted by curves of the form of Equation (11), if $K = 2.80$ and $u = 1.5$, the deviation in all cases being well within the range of observational errors of the gage readings, which were made on gages marked on the walls of the approach channel, without any provision for stilling. This constant value of the coefficient and of the exponent offers much more promise of accurate extrapolation for the application to larger rating flumes than Equations (7), in which both the coefficients and the exponents vary with the size of the flume. Expressed in the same manner as Equation (11) formulas for a gage 3 ft down stream from the transition may be written from the following:

Width of flume, in feet	K	u
1	6.35	1.321
2	6.52	1.277
3	6.62	1.245

The extrapolation of these formulas up to widths of 50 ft or more, such as may readily be required, may easily introduce serious errors.

One of the causes for this observed variation in coefficient and exponent, of course, is that the stilling-well connections in the three sizes tested were at relatively different distances down stream from the flume entrance, this distance being 3 ft in each of the flumes tested. Had the stilling-well connection been made 3 ft below the entrance on the 1-ft flume, 6 ft on the 2-ft flume and 9 ft on the 3-ft flume, making each flume a model of the others, it is possible that the curve formulas might have shown better agreement for the three sizes.

In the three small rectangular flumes originally tested at the San Dimas Experimental Forest, the approach channel had a flat grade and was three times as wide as the rating flume itself. In all the runs reported by the authors the flow was at depths greater than critical in the approach channel and at depths less than critical in the lower end of the rating flume, passing through critical depth at some point between the transition section and the stilling-well connection section. Obviously, if the rating flumes were long enough, normal flow at less than critical depth would eventually be established at some distance below the transition section, this distance depending on the rate of flow. For very low flows in the 1-ft flume, normal flow would be established in the rating flume at, or up stream from, the stilling-well connection. In such a case the ordinary formulas for uniform flow would apply. If the rating curve for a rectangular flume of uniform cross-section and slope is computed by the Manning formula, it will be found to be fitted closely by the formula,

$$Q = K_1 \, b \, H^{1.5} \qquad (12)$$

which, except for the coefficient, is the same as Equation (11) applying to gage heights observed in the approach channel. This coefficient will be found to depend only on the grade and roughness of the channel, wherever normal flow exists.

It seems, therefore, that the rating curve for a gage located 3 ft down stream from the lower end of the entrance transition section cannot be expressed as a simple parabolic function for the entire range of gage heights. It follows that the method of least squares, involving the assumption of expressions having constant exponents, cannot safely be used in computing the rating curve for a gage located in that part of a stream where the flow is accelerating. This point was illustrated recently in rating a 10-ft flume of the San Dimas type in actual use, when it was found that the discharges, when plotted against gage heights in the rating flume on logarithmic paper, could be fitted by a curved line. The extension of this curved line to higher stages than covered by the discharge measurements showed a wide departure from the straight line derived by the method of least squares applied to the discharge measurements.

The statement is made by the authors that the new design is "unaffected by velocity of approach," (Conclusion (4)) referring to the San Dimas type of rating flume. This statement must be incorrect, since the velocity of approach has a marked effect on the entrance losses and on the rate of acceleration in the rating flume at the stilling-well connection. If the approach velocity were zero, the velocity at the stilling-well connection would be in the zone of acceleration from zero velocity to normal velocity for a flume having a 3% grade. It should be expected, therefore, that a higher velocity of approach, such as would result from decreasing the width of the approach channel, would cause greater velocities at the stilling-well connection for a given rate of flow. This would mean smaller depths and larger discharge coefficients for the rating flume. That this is true has been demonstrated by recent experiments in which the width of the approach channel has been reduced.

Since the velocity of approach has a marked effect on the rating of the San Dimas type of rating flume, it follows that changes in the grade of the approach channel may cause changes in the flume rating. The experiments at the San Dimas Experimental Forest were made with an approach channel having a flat grade. Under these conditions the flow in the approach channel was at depths greater than critical during all the runs reported. In the ordinary flood channels in which some form of rating flume or artificial rating control is required if reasonably stable ratings are desired, the normal slope which has been established by the stream itself is commonly as great as 6% or 8%, the grade depending on some relation between the size of the stream and the nature of the débris being transported by the stream. In such a flood channel the velocities, even at low stages, are usually super-critical, and the depth of flow may be considerably less than critical.

If such a flood channel should be obstructed by a rating flume of the San Dimas type in which the width of the rating flume is one-third that of the approach channel, a hydraulic jump would probably occur at some point above the rating flume entrance, at all except low stages. Under these conditions, for clear-water flow, the rating of the flume might be expected to approximate that determined by the experiments reported by the authors, if the stilling-well connection is situated at a distance down stream from the transition bearing the same relation to 3 ft that the width of the flume bears to the width of the experimental flume whose rating curve is used.

However, with débris-laden stream flow, the conditions at the entrance to the rating flume should be expected to change radically after the construction of such an obstruction as this rating flume would constitute. In such a case any débris picked up by the stream in its upper reaches by the natural processes would be transported by the stream to the point of formation of the hydraulic jump. At this point a typical débris dune would be deposited, which would travel up stream or down stream in the manner typical of traveling débris dunes in flood channels. Eventually, as the supply of added débris became less than the transporting capacity of the stream, the dune would move down stream to the entrance of the rating flume, and would establish itself there with a down-stream face having the normal slope, depending on the relation between the rate of flow and the nature of the débris.

The establishment of such a débris dune at the entrance to the rating flume would tend to cause a change in entrance velocity similar to that caused by narrowing the approach channel, and a corresponding increase in rating-flume discharge coefficient should be expected.

In the experiments at the San Dimas Experimental Forest, débris was introduced by men with wheel-barrows, and at no time while the experiments were being observed by the writer did the dune fully establish itself at the entrance to the rating flume. Under the conditions of those experiments a small quantity of débris was cut away from the edges of the dune by the stream and carried through the flume, which kept itself clean. At no time, however, was the quantity of débris being transported through the rating flume more than a very small proportion of the débris-carrying capacity of the stream in the rating flume. Under these conditions it is not surprising that no serious change in flume rating was observed. To conclude from these experiments that, under natural conditions, débris would have no serious effect on flume rating is by no means safe.

In the case of the 10-ft San Dimas type of rating flume a small landslide occurred a short distance up stream some time after the flume was installed. Together with material dumped in the stream bed during the construction of the flume, this landslide formed a débris dune which was gradually moved down stream by the subsequent small flows of clear water. At the time a test was conducted for the purpose of determining a rating curve for the 10-ft flume the dune occupied a position about 25 ft up stream from the rating-flume entrance. The comparatively large flows used in the early part of this test moved the débris dune rather rapidly down stream until it occupied a position just above the rating-flume entrance. In this position it had rather a marked effect on the nature of the flow through the flume, as it caused the water to enter at an angle with the center line, resulting in cross-currents; and, at times, water "piled up" against the side of the flume opposite the stilling-well connection. Under these conditions it was not surprising to find variations of 10% or more in successive determinations of the stage-discharge relation. At the time the tests were concluded a small quantity of débris had begun to be discharged through the rating flume. At no time during the test was additional débris being added to the stream by further landslides above the rating flume; nor was the quantity of débris being carried through the flume, any appreciable percentage of the flow of clear water.

For the purpose of studying the probable behavior of this 10-ft flume under conditions of larger continuous landslides up stream from the rating flume, the writer constructed a small model and made qualitative studies of its behavior. In these studies the débris used was in the form of fine wind-blown beach sand taken from the back of a large sand dune about one-half mile back from the ocean shore. It consisted of irregularly shaped grains varying in size from about 0.001 in. to 0.020 in. and had a considerable degree of dimensional similitude with the actual débris in the prototype.

In the earlier part of his model tests the writer duplicated as closely as possible the conditions during the period leading up to, and including, the aforementioned rating tests. During this part of the model tests the appearances of conditions in the rating flume were about as previously observed in the prototype.

In the second part of the model tests a large volume of additional débris was introduced in the form of landslides some distance up stream from the rating flume. Under these conditions the approach channel gradually established itself at a uniform grade of about 8% leading down to the rating-flume entrance. This part of the test was concluded when the up-stream face of the débris dune reached the edge of the landslides. During the entire period of this part of the model tests some débris was rolling down the down-stream face of the dune and out through the rating flume, but at no time did the débris passing through the rating flume exceed 1% or 2% of the volume of clear-water flow, and the rating flume remained clean.

In the third part of the model tests a large volume of additional débris was introduced in the form of landslides occurring on top of the down-stream face of the large débris dune formed in the second part of the test, but still at a considerable distance up stream from the rating flume. Under these conditions the toe of the dune moved down into the rating flume to a point down stream from the stilling-well connection and the volume of débris passing through the rating flume increased to about 20% or 25% of the volume of clear-water flow. As the flow of water was decreased to zero, débris remained deposited in the rating flume, and the stilling-well was put out of commission. During this entire part of the model tests the rating flume obviously failed to function as a stable rating device.

These preliminary model tests indicated that the San Dimas type of rating flume is far from being the answer to the engineer's prayer for a stable rating device for shifting flood channels. Further quantitative tests are necessary to determine, first, how far this type of rating flume is from being the desired answer; and second, to find out, if possible, what type of rating device is best adapted to the conditions that must be met in the flood-control and flood-measurement fields. Experiments along this line should be based on the assumption that the best type of rating device will be that which least disturbs the balanced conditions that Mother Nature herself establishes in flood channels.

The most stable natural "controls" for a gage on a débris-laden stream are those in which, at all stages, there is a very slight increase in water-surface slope a short distance down stream from the gage. At such gaging stations there is a very definite relation between the exponent in the rating curve formula and the shape of the cross-section of the stream at the point where the change in slope of the water surface occurs.

The writer gratefully acknowledges the assistance he has received from members of the Los Angeles County Flood Control District, and from the authors themselves, in the preparation of this discussion. Unpublished water-surface elevations in the approach channels were made available through the courtesy of Professor Kotok.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

WATER TRANSPORTATION VERSUS RAIL TRANSPORTATION

A SYMPOSIUM

Discussion

BY H. R. FAISON, M. AM. SOC. C. E.

H. R. FAISON,[22] M. AM. SOC. C. E. (by letter).[22a]—Opportunity to take part in the discussion of water transportation costs in an impartial technical publication is welcomed by the writer. After all, finding the value of improvements for navigation is a technical problem. Most of the publicity accorded this important topic in the commercial press is too partisan in viewpoint to serve the man who is seeking facts. The "picture" is too likely to be colored, perhaps unconsciously, by the interest of some particular section, industry, or type of carrier; or distorted by an array of selected statistics which illuminate only one side. As a consequence some theories as to the cost of water transportation, not strictly in accord with sound economics, have apparently gained wide popular acceptance even among engineers. In spite of Mr. Wonson's obvious sincerity, some of these theories leading to unsound conclusions seem to have crept into his otherwise able and fair treatment of the subject. With his basic principles the writer is in complete accord.

The writer does not advocate providing or maintaining, by artificial stimulants, any route or means of transportation that proves uneconomical. There would be no point in trying to hold traffic on the rivers to keep water service alive if no general public benefit resulted. He would make an even broader application of this principle to cover outmoded and uneconomical transportation routes and service of all kinds in which the public has a stake. Differences with Mr. Wonson arise, not as to principles, but as to methods of appraisal and bases of comparison between the several types of transportation. It seems in-

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NOTE.—The Symposium on Water Transportation Versus Rail Transportation was presented at the meeting of the Waterways Division, Little Rock, Ark., April 25, 1936, and published in September, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: October, 1937, by George Hartley, Esq.; December, 1937, by W. D. Faucette and J. E. Willoughby, Members, Am. Soc. C. E.; January, 1938, by C. D. Bordelon, Esq.; and February, 1938, by J. E. Goodrich, Esq.

[22] Senior Engr., Board of Engrs. for Rivers and Harbors, Washington, D. C.

[22a] Received by the Secretary January 28, 1938.

cumbent upon engineers confronted with waterway problems for solution in the interest of the people as a whole to exercise particular care in adopting the proper "yardstick." Because there are well known inherent draw-backs to barge transportation, it cannot soundly be assumed that all inland waterways are bad investments. Averages obtained by grouping good and bad projects together are obviously valueless. It is frequently found in the case of individual improvements that favorable factors, fully recognized and evaluated, outweigh all natural disadvantages.

Circuity.—Prominent among such disadvantages stressed by Mr. Wonson is the fact that river routes are generally longer than competing rail routes. Like most of the other elements of comparison, circuity in the case of individual waterways can usually be measured and given a definite value. Mr. Wonson attempts to do just that, but he derives his circuity factors from a comparison of river mileage with the shortest rail distance, ignoring competing rail routes. He uses these factors to raise the actual unit cost of freight movement by water to a higher sum for comparison with rail cost. This procedure gives no consideration to the fact that the great preponderance of rail freight also travels by circuitous routes. The Federal Coordinator of Transportation finds that the average rail carload moves 11% farther than would be necessary by a direct line in common use.[23] As a result of their analysis of freight movements by rail between river ports, the barge lines have petitioned the Interstate Commerce Commission to recognize officially the rail circuity factor of 20% in excess of short line mileage in the Mississippi and Ohio Valleys.[24] In cases of competition with water routes involving relief from the long-and-short-haul requirements of the Transportation Act, it is not uncommon to authorize rail routes as much as 50% longer than the short line. It should be remembered that the circuity of some of the principal inland water routes is less than that. The canalized Monongahela River is actually shorter, as a matter of fact, than competing rail lines, but the fact is dismissed in Mr. Wonson's analysis as negligible. Obviously, the circuity factors in the paper under discussion must be reduced to conform to the facts, if the reader is to arrive at the correct answer.

Amortized Expenditures.—It was not found possible to reconcile Mr. Wonson's distribution of costs among the several interests affected with published statistics. Apparently he charges such items as bank protection, and, in fact, all the money ever spent on inland streams except flood control allotments, to existing navigation facilities. According to his theory these charges must be borne by existing water lines on their present tonnage or improvements should be abandoned. In this situation the engineer must face the fact, squarely, that the millions spent for purposes other than navigation, and for navigation improvements which have been outmoded and abandoned or superseded by modern projects, are not involved in any conceivable way with his problem.

Only a small part of the open channel work done on the Ohio and Warrior Rivers, for example, contributed in any manner to the present efficiency of these waterways, but it served its purpose for many years and its cost was amortized by contemporary river commerce. At most, only the cost of useful work necessary for present navigation has any practical bearing. On the Warrior River, canalization was begun in the Nineties and completed just prior to the serious congestion of the Nation's transportation system incident to the World War. Justification of expenditures providing this alternative freight route between an essential producing area and tidewater would scarcely have been seriously questioned at that time. Such waterways cannot be duplicated over-night, and naturally it is impossible to expand and contract permanent investment already made, to synchronize with fluctuating traffic volume and business cycles. In this connection it is far from clear just what end would be served with respect to all these past expenditures if the waterways were to be abandoned now as recommended in Mr. Wonson's paper.

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[23] Freight Traffic Rept., Paragraph 71.

[24] Interstate Commerce Commission, Docket No. 26712.

Interchange of Traffic.—Mr. Wonson's analysis indicates that benefits due to the improvement of inland rivers are restricted to communities along their banks. The evident intent of all legislation having to do with water transportation is to secure the widest possible distribution of benefits paid for by the general public. The clause of the act establishing rail-water differential routes and rates is obviously designed to extend to "dry land" territory the same scale of savings enjoyed by ports along the waterways. Barge lines have in fact instituted a broad system of traffic interchange with railroads and motor truck lines extending far into the riverless regions of the country. The effect would doubtless be felt more generally were it not for the openly avowed unwillingness of the railroads to co-operate. As a group, their opposition has ranged from challenging the validity of the legislation in the Courts and opposing each proposed extension of the system, to the current proceeding before the Interstate Commerce Commission aimed at cancellation of all routes and differentials through which localities inaccessible by water already enjoy the benefits of barge service.[24] It is desired to emphasize at this point that the writer does not mean to imply that there is anything fundamentally wrong with this perfectly natural attitude of rail and water competitors toward each other, their interests being involved primarily in the profitable operation of their individual properties. The engineer must recognize, however, that these interests cannot be expected to coincide with each other nor with the interest of the country at large.

Distribution of Benefits.—For reasons obscure to the writer, Mr. Wonson seems to hold that savings resulting from common carrier water service may be counted as public benefits, whereas savings from contract or private water carriage are not public benefits. Congress does not attempt to assess the benefits resulting from the various types of river improvement to any particular region or group of beneficiaries, except where the effects are local in character. The waterways are made open to all, and because of the competitive element it is impracticable for the producer, carrier, or distributor of goods to monopolize the benefits. The writer has long recognized the futility of trying to trace savings from river improvement to their ultimate recipients, or to devise any system of charges that would distribute the burden more equitably than the present one in which the public pays for its waterway and the shipper pays for transportation on it. He agrees with Mr. Wonson that improvement for navigation is not warranted if the potential aggregate of such costs is greater than by other available means of transport. He differs from him in the method of measuring these costs, and the significance of the results. The engineer's problem is not to identify the beneficiaries, but to determine whether there is, in fact, a net economic saving to be distributed.

Tonnage Estimates.—Mr. Wonson attaches great significance to the relative public use of rail and water facilities. On the other hand, his estimates of the volume of water commerce that can reasonably be expected are based on the comparatively small tonnage which the boat lines thus far have been able to develop and hold in the face of competitive rail rate depression. It is necessary to bear in mind that inland river improvements comprise a system that is still under construction and far from complete. Records show[25] that the growth of water commerce is in the direction of longer and longer hauls as the individual waterways are progressively extended and connected into a system. The period for building up permanent water traffic involving the gradual development of storage warehouses, grain elevators, cotton compresses, industrial plants, and auxiliary services at the water side without sacrificing existing facilities, is a period of evolution reasonably expected to extend over a considerable number of years. It is believed that the greatest impetus toward building up a representative volume of tonnage on waterways will result from the eventual adjustment of freight rates to conform more closely than they now do to the cost of the transportation service. That this trend is officially favored is evidenced by numerous decisions of official regulatory agencies; for example, a concurring opinion of a member of the Interstate Commerce Commission in the Petroleum Products case,[26] reads in part as follows:

> "I am much inclined to believe that a system of rates for all agencies based on cost plus a reasonable profit under normal traffic conditions would allow them all to perform their appropriate functions, and in the end be better for all concerned, including the country and its industries, than any system of competitive rates."

Mr. Wonson's insistence that freight charges measure the cost of the service, and his attack on waterways as tending to favor regions adjacent to rivers at the expense of more inaccessible territory would lead the reader to infer that rail freight charges are commensurate with cost, and that rail carriers do not tax riverless regions with high rates in order to support low rates along the waterways. The writer agrees that when rates are made proportionate to cost of operation, it is reasonable to expect an appropriate distribution of traffic among the several forms of transportation, to each falling the share best adapted to its particular kind of service. Direct comparisons of tonnage would then be significant, and total relative ton-mile costs would offer conclusive evidence of the degree of justification of river improvements. That ideal condition is more remote from the rail rate structure, however, than from the scales prevailing on water.

In its profound effect on relative public use, it is necessary to recognize the practically universal railroad policy of depressing rates to profitless levels when confronted with competition from water lines. Such rates not only fail to cover fixed charges but usually cover only a part of operating and maintenance expense.[27] The expectation of this result on the part of shippers undoubtedly gives rise to much of the organized agitation for improvement of inland streams, and introduces the most troublesome single factor into the problem. The writer joins Mr. Wonson in earnestly deploring this use of waterways to regulate rail rates, but the depression of rates to meet water competition, of course, is a wholly voluntary act on the part of the railroads. It is clear that freight recaptured by this means from economical routes and diverted to uneconomical routes, causes the public to forego the real savings made available by water. When thus deprived of profitable tonnage the barge lines sustain material loss which is totally uncompensated by profit to the rail carrier or real benefit to the shipper.

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[25] See Tables, Annual Repts., Chf. of Engrs., U. S. Army, 1927 to 1936.

[26] 194—ICC—31.

The Added Traffic Theory.—For the most part the competitive depression of rail rates is confined to traffic between ports served by waterways, the rates between intermediate and interior points being held up on the "dry land" level. In addition to this non-competitive traffic between intermediate points, the railroads normally handle between ports many commodities not adapted to barge movement. Under these circumstances the depression of rail rates to competitive levels on commodity movements adapted to water theoretically, at least, leaves the bulk of rail traffic to carry the expense burden. Obviously, the total revenue collected by either type of carrier must exceed its total expenses "in the long run" if it is to prosper. Thus, the water carriers would be completely at the mercy of rail competition were it not for the fact that under normal conditions barge lines can handle adapted commodities at much less cost than rail carriers.

The "added traffic" theory invoked by the railroads to meet this situation is based on the premise that once a carrier is provided with adequate equipment and terminals, and is in operation, with supervisory staffs employed, and a sufficient volume of traffic moving at normal rates to pay the overhead, the out-of-pocket cost of handling any additional freight that may be secured is but a small part of the full cost of maintenance and operation. For the creation of new traffic there is nothing fundamentally unsound in this theory, provided always that rates are so fixed that the extra tonnage brings in as much money as is spent to move it. It is when carriers begin to view each other's business as potential added traffic, and to prey upon each other's basic tonnage by means of out-of-pocket rates, that the theory breaks down and the public eventually has to make good an uncompensated loss. The raided competitors certainly cannot afford to submit cheerfully to "subtracted" traffic, nor subscribe to the theory that their revenues and expenses are not affected thereby. When carried to the extreme now characterizing the aggressive campaign against water transportation, almost all traffic is rapidly taking on the status of added traffic, and revenue with which to pay full operating, maintenance, and fixed charges, not to mention dividends, is becoming progressively scarcer among the railways adopting this policy.[28]

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[27] ICC Stats., 3 352—1933.

[28] See testimony of the rail representatives in Ex Parte 123, ICC, December, 1937.

Practical Illustration.—The most typical case brought to the notice of the writer was decided July 23, 1937,[29] and involves the shipment of about 100 000 tons of molasses annually from the New Orleans area to Peoria, Ill., on the Illinois River. When this movement began the rail rate was $6.00 per ton, which over the short rail line of 840 miles would yield 7.1 mills per ton-mile. The industry purchased a towboat and tank barge equipment and found that it could transport its own raw material for $2.60 per ton, or at 2.0 mills per ton-mile for a water distance of 1 270 miles. The initial floating plant being found inadequate, the shipper was negotiating for the purchase of an additional fleet when the railroads depressed their rate to $3.50 per ton. This reduction was sufficient to hold a substantial part of the business for the railroads and to bring about the cancellation of orders for new barge equipment. The reduced rate yields 4.2 mills per ton-mile gross. This rate compared with the 9.9 mills average rail rate used by Mr. Wonson would indicate that the molasses business to Peoria is not "pulling its weight" on the railroad but that the river loses tonnage it could carry profitably. The rates on molasses to intermediate dry-land points, of course, are held up to the old high level and, together with other non-competitive traffic, are doubtless carrying the added burden. This typical case is used to emphasize the fact that each departure from the long-and-short haul regulations of which there are hundreds, tends to divert a substantial block of tonnage from the waterways where it could be handled economically, and where it would serve to decrease the average unit costs of water movement.

Effect of Rate Cutting.—No single type of carrier holds a vested right in any particular traffic. As already stated there would be no point in trying to hold traffic on the rivers to keep water transportation alive if no net benefit resulted. However, water transportation is clearly entitled to a chance to justify itself, before the public can pass judgment intelligently. There is no logic in recapturing tonnage from water by means of profitless rail rates, and then pointing to the lack of water-borne tonnage as significant. Out-of-pocket rail rates hold the volume of barge tonnage far below that which would normally seek that form of carriage, and which river carriers have justifiably equipped themselves to handle. Naturally unit costs of water line operation rise in response to such depletion, and the cost per ton to the Federal Government of waterway construction is unduly inflated. It will be readily understood that, in use of the term, "inflated," and all other expressions of difference of opinion with Mr. Wonson throughout this discussion, there is no thought of imputing any conscious or deliberate bias. The writer believes that the angles stressed herein have been overlooked, and that their full consideration will reverse some of Mr. Wonson's conclusions. He is not concerned with regulation of the practices of the carriers, but with due recognition of their effect on cost factors.

Summary.—Long-range analysis of the future of river commerce must contemplate, in all reason, a closer approach to the cost basis of fixing transportation charges than now prevails. The present system is leading toward bankruptcy for the carriers and a collapse of the transportation structure. Doubtless there is a present surplus of transportation capacity in certain areas and the least efficient and economical carriers, whether rail or water, must inevitably perish, to the advantage of the general situation. It seems scarcely reasonable to the writer, however, to hold that the internal waterway system is a liability rather than an asset to the country. Eliminating the cost of improvements for other purposes such as prevention of soil erosion and the cost of outmoded and superseded improvements, placing the remaining costs on a factual basis, and using the low operating costs of going barge lines, completely changes the picture in comparison between rail and water carriage.

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[29] 223—ICC—221.

It is firmly believed that traffic, if permitted freely to seek the best adapted routes, will flow in greatly increased volume to the important links in the waterway system, resulting in total ton-mile costs so reduced as to compare favorably with the most economical competitive service. Lest it appear that the points discussed herein are trivial in their bearing, the reader is asked to make a hasty computation of the results of applying Mr. Wonson's method of appraisal. A 50% drop in water-borne tonnage as a result of rate cutting, means a 100% inflation of the cost of Federal improvement, computed on the ton-mile basis, in addition to the rise in unit costs of the boat lines. Adding extraneous costs, and those for superseded improvements, using cost ratios of projects still under construction and not yet ready for traffic, employing excessive circuity factors, all err in the same direction, combining to depress still further the water-borne tonnage figures and inflate unit costs. If taken at their face value, these factors would nullify every real economy the most worthy waterway project is capable of effecting; and yet it is this depleted tonnage and these inflated costs that are cited as proof of the failure of river improvement to justify itself as a national policy.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

THE DESIGN OF ROCK-FILL DAMS

Discussion

BY MESSRS. HOWARD F. PECKWORTH, OREN REED, WALTER L. HUBER, SAMUEL B. MORRIS, AND L. F. HARZA

HOWARD F. PECKWORTH,[12] M. AM. SOC. C. E. (by letter).[12a]—The definition of rock-fill dams, as given by Mr. Galloway, does not include those with a facing of impervious rolled-earth fill. It is perfectly evident, from a historical viewpoint, why this was done but the writer believes that a rock-fill dam with an impervious rolled-earth blanket should come under the heading of rock-fill dams especially as the rock-fill in both cases is serving the same function—that is, as weight to support the impervious section. Naturally, the grading of the rock behind the impervious section may be different, but the materials and functions of the rock-fill are the same in both cases. Mr. Galloway's paper is especially useful in defining the basic principles of design for this type of dam as developed in the West because there is no doubt that in the near future this type will be used in many variations in connection with rolled-earth fill in the South and East as well as in the West.

At present, a graded sandstone supporting an impervious rolled-earth blanket is being used in the construction of Inland Dam, near Oneonta, Ala., for the City of Birmingham. This is one case of the use of sandstone, although not in the exact type described by Mr. Galloway; and as sandstone will undoubtedly be used some day in the type described, it is of interest to note the following characteristics of a sandstone quarry:

(1) The line of demarcation between the over-burden and the stone may be very indistinct, the over-burden grading gradually from top soil through clay, sandy clay, and decomposed or unconsolidated sandstone to good, hard, suitable sandstone. The question will arise as to exactly where suitable stone begins and many arguments can be avoided by including in the specifications, "suit-

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NOTE.—The paper by J. D. Galloway, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. Cecil E. Pearce, and H. B. Muckleston; January, 1938, by Harold K. Fox, M. Am. Soc. C. E.; and February, 1938, by Messrs. Charles H. Paul, and A. Floris.

[12] Res. Engr., Birmingham Industrial Water Supply Comm., Birmingham, Ala.

[12a] Received by the Secretary January 27, 1938.

able stone is that which rings under a hammer or passes a compression test of (state allowable)."

(2) In a sandstone quarry there is likely to be an excess of quarry waste; decomposed or unconsolidated sandstone, broken down by blasting to sand or fines; and included with this material may be horizontal seams of shale or vertical seams of clay or top soil. In the quarrying operations this must be expected and must be taken care of by suitable methods of wasting.

The writer checks Mr. Galloway's ratio of 1.3 to 1 for free falling sandstone rock, 3 in. to 1 in. in size, and for sandstone screenings, 0.5 in. to zero. He also checks the values, 1.3 to 1 and 1.4 to 1, for free falling sandstone rock, 15 in. to 3 in. in size.

OREN REED,[13] M. AM. SOC. C. E. (by letter).[13a]—As stated by the author, the design of rock-fill dams was developed primarily in California. Experience has established certain rules of design, which have been used in the construction of several notable structures such as the Salt Springs Dam, in California. Economy by the use of native materials is ordinarily the controlling factor in determining the selection of a rock-fill dam for any site.

Rock-fill dams are not common in Europe. However, there are a few structures of this type, one of the largest being the Porjus Dam, in Sweden, which was inspected by the writer in 1930. The dam at Porjus, 30 miles north of the Arctic Circle, consists of an earth and rock-fill, two concrete spillway sections over the natural channels, and a short concrete gravity section, giving a total length of 4 100 ft. The dam, completed in 1912, was constructed largely of earth and rock-fill because of the high cost of concrete in this remote location. Like the Crane Valley Dam, in California, the section consists of a rock-fill down stream from a center core-wall of reinforced concrete, and gravelly earth-fill, covered with rip-rap, on the up-stream side. The down-stream face was built on a slope of 1 on 2 and the up-stream slope varies from 1 on 2 at the top to 1 on 3 at the toe. The crest width is 15.4 ft. The normal free-board is 5.2 ft. The material for the rock-fill was obtained from the excavation for the spillway and intake tunnel.

The two spillway sections have a total length of 648 ft, each consisting of concrete arches between concrete buttresses. The crest of the spillway sections is regulated by timber needles 15 ft long, supported by hinged steel bents, and a roller gate 16 ft in diameter and 39 ft long. The roller gate is remotely controlled from the power-house, providing close regulation of the pond level for smaller fluctuations of flow.

Due to better transportation facilities and increased knowledge of concrete construction, a multiple-arch dam was given preference when the regulation of the Sourva Lakes was under consideration. The dam at Sourva provides regulation on Lule River to increase the normal low-water flow at Porjus. This dam, completed in 1924, has a maximum height of 75 ft and a length of 785 ft.

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[13] Asst. Constr. Engr., TVA, Pickwick Dam, Tenn.

[13a] Received by the Secretary February 5, 1938.

Two more recent rock-fill dams in Sweden are situated at Sillre and Torrön. The dam at Sillre, completed in 1933, has a length of 920 ft and is constructed of rock from the power tunnel excavation. The dam is located on a marshy and morainic foundation. The down-stream face has a slope of 1 on 1.5 and the up-stream face, a slope of 1 on 1.6. The crest width is 9.85 ft, and the maximum height is about 26 ft. The water-tight part consists of stiff puddled clay, placed on the sloping up-stream face of the dam. The puddled clay has a normal thickness of 2.0 ft at the crest of the dam and 4.9 ft at the foundation. The clay blanket is joined at the bottom to a sheet-piling cut-off, which extends 13 ft to 16.5 ft in the hard, but pervious, foundation. The clay blanket has an up-stream slope of 1 on 2 and is separated from the rock-fill by a thin layer of gravel, which acts as an inverted filter. An insulating layer of peat was placed on the face of the clay blanket and heavy rubble rip-rap was constructed on the insulation material to prevent erosion by wave action. The development at Sillre is a pumped-storage project and the capacity of the reservoir is large in relation to the run-off. There is no spillway, but a sluice-way is provided which is controlled by a slide gate, 4 ft by 9 ft in section.

The dam at Torrön, completed in 1936, is a true rock-fill with a timber facing, which is fastened directly to the rock of the rubble wall. The up-stream face was built on a slope of 1 on 1, whereas the down-stream face has a slope of 1 on 1.5. The crest width is 10 ft. The timber deck was selected instead of a concrete deck because it was expected that settlement of the rock-fill might be great enough to cause dangerous cracking of a concrete deck. A low concrete cut-off wall is provided to connect the timber facing to the rock foundation.

The writer considers that a well-designed rock-fill dam is a relatively stable structure and in locations where cold temperatures are experienced, a fill dam may be preferred to a concrete structure.[14] In some cases, as noted previously, the spillway may be provided with gates with perfect safety. However, spillway regulation should be of a positive type and operated by experienced crews.

WALTER L. HUBER,[15] M. AM. SOC. C. E. (by letter).[15a]—At least a partial remedy for the lack of any published compilation of authentic data on rock-fill dams is found in this paper. The rock-fill dam is a type of structure which has been widely used in the western part of the United States. The author has confined his discussion to true rock-fill dams which, as he states, consist of three elements: "A loose rock-fill forming the mass of the dam; an impervious face next to the water; and a rubble cushion between the two." From this true type many variations have been developed, including important structures built in recent years wherein a very substantial section adjacent to the impervious face is of rolled fill, back of which is a loose graded rock-fill (such as San Gabriel Dam No. 1 of the Los Angeles County Flood Control District, with a maximum height exceeding 350 ft). As is indicated in the paper, the true rock-fill type is the result of an evolution from the rock-filled log crib type and from an intermediate type wherein the section was largely of uncemented, hand-laid, rubble masonry.

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[14] "Preventing the Disintegration of Dams," by Oren Reed, *Civil Engineering*, January, 1933.

[15] Cons. Engr., San Francisco, Calif.

[15a] Received by the Secretary February 5, 1938.

The author mentions the Bear River and Meadow Lake Dams, on the Mokelumne River, in California, built from 1900 to 1903, as about the last rock-fill dams to have the down-stream face protected by a dry rubble wall. Attention is directed to the Utica Dam, of the Utica Mining Company, on the head-waters of the North Fork of Stanislaus River, in California, which was completed to a height of 60 ft in 1908 and which has a back face of dry rubble.

Mention is made of Sabrina and Hillside Dams, on Bishop Creek, California, both of which are of the true rock-fill type, 70 and 80 ft in height, respectively. Fig. 3 is a view of the timber face of Sabrina Dam after the rock-fill had acquired most of its settlement. This is in no way indicative of the present condition of the structure. As the author has noted, the timber face was replaced after twenty years of service. Fig. 14 is offered to show the present condition of this face. The original facing was of native lodge-pole pine timber, admittedly an inferior grade of lumber, but because of the inaccessibility of the site of both Sabrina and Hillside Dams from 1908 to 1910 this was used for reasons of economy. The timber face was not laid directly on the loose fill as noted by the author, but the cushion of hand-placed rock was comparatively thin.

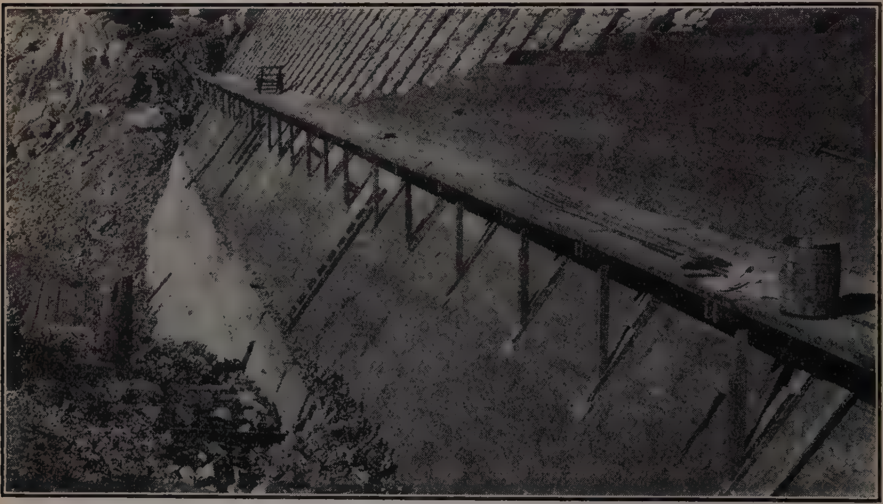

FIG. 14.—REFACING SABRINA DAM, OCTOBER, 1929

After twenty years of service, the original timber faces of both dams were showing signs of deterioration and leakage was increasing. Accordingly, these faces were removed and, in each case, the cushion of hand-placed rock was brought up to an even surface. Pressure-creosoted, 8-in. by 8-in. Douglas fir vertical stringers were placed in the reconstructed face of the rock cushion at intervals of 5 ft 4 in. and securely anchored by 1-in. round steel anchors which were spaced at 6-ft intervals and secured to sections of rails embedded in the dam. Directly across the stringers is a plank facing of redwood which forms the water-tight element. These planks are 3 in. by 12 in. Redwood splines, 0.5 in. by 1 in., are placed in the horizontal joints, and the end joints are splined with $\frac{1}{8}$-in. by 2-in. galvanized flat steel splines. The redwood facing varies from three layers of 3-in. planking at depths exceeding about 38 ft to a single layer for a depth of less than about 11 ft. Fig. 15 shows the face of Hillside Dam when the work of resurfacing was practically completed in September, 1930. It is believed that the life of the new faces constructed of redwood and pressure-treated Douglas fir will greatly exceed that of the original faces constructed of native timber of inferior quality. It is interesting to note that estimates indicate that reconstruction of timber faces, as required from time to time, is more economical than the construction of a so-called permanent face under the difficulties imposed at the sites by inaccessibility and by short seasons due to high altitude.

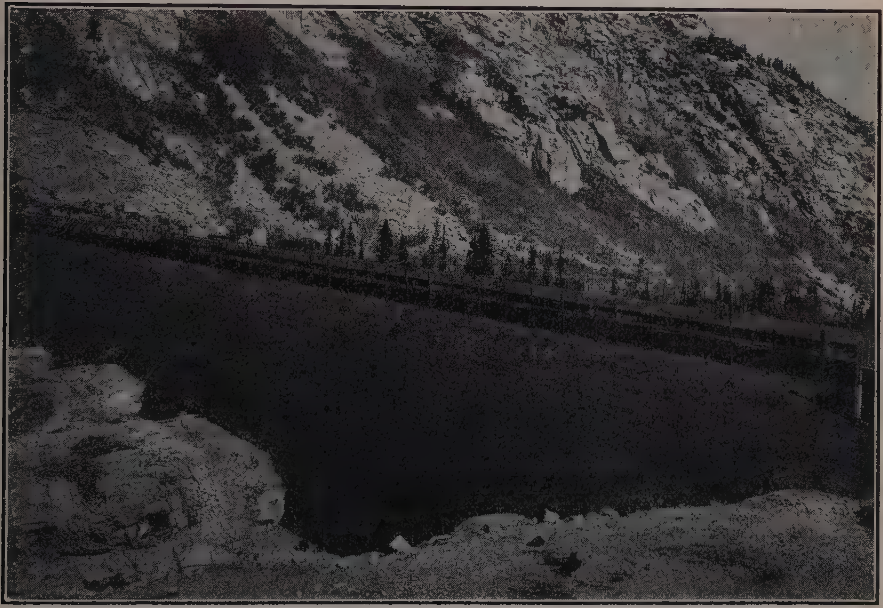

Fig. 15.—Refacing of Hillside Dam Practically Completed, September, 1930

Another advantage secured is that the flexibility of the original timber faces allowed them to follow the unequal settlement of the rock-fills and, at the same time, remain comparatively water-tight. The new faces have been applied since the settlement is substantially completed and these facings will not be subjected to the same warping. Although this type of facing has advantages for dams of lesser height, the designers of some of the very high rock-fill dams have sought a facing of more permanence and have applied it with the original construction. In each instance a degree of rigidity has resulted which has given some, although not always serious, trouble when the inevitable settlement of the rock mass occurred. In the case of San Gabriel Dam No. 2, of the Los Angeles County Flood Control District, a timber face has been installed as a temporary expedient after settlement of the rock-fill had seriously distorted the laminated concrete slab face.

A disadvantage of timber facing is the possibility of destruction by fire when the water is drawn down in the reservoir. Serious damage from this source at Sabrina Dam was traced to the carelessness of a fisherman. The owners have since enclosed all their timber-faced dams with high and substantial wire fences with locked gates.

Samuel B. Morris,[16] M. Am. Soc. C. E. (by letter).[16a]—The historical development of the rock-fill dam has been well presented in this paper and the author has outlined the general criteria for its design and construction, to the present maximum height of slightly more than 300 ft. There is one aspect relative to rock-fill dams with which the writer has long been concerned. How will the average, well-constructed, high, rock-fill dam behave during violent earthquake shocks? Earthquake accelerations up to 0.1 g are generally being applied to dams, school buildings, and other structures in regions where earthquakes of such magnitude are likely to occur.

More recent designs of concrete or masonry dams of all types in California and other parts of the United States liable to earthquakes have made provision for resistance to earthquake shock. Such designs are now made to provide for two principal types of forces acting upon a dam during such shock: (1) The forces due to acceleration acting upon the mass of the dam; and, (2) the hydrodynamic forces resulting from the motion of the dam against the still water of the reservoir bearing upon its up-stream face.

There is probably no type of structure built by Man which offers so great a potential hazard to life and property as a large dam and reservoir up stream from a heavily populated area. Proper consideration of safety is paramount. In connection with the design of a high dam so situated the writer, several years ago, took occasion to visit a large number of existing dams and to study different types from the standpoint of safety in a region where earthquakes are to be expected.

The rock-fill dam has been suggested to the writer by some engineers as being particularly suitable for use in regions liable to earthquakes, on account of its large mass and flexible character. The writer has been inclined to question the behavior of high rock-fill dams during earthquake shocks.

The settlement of fills during earthquakes has been general observation and common knowledge. Railway and highway fills, particularly those at bridge approaches, have settled substantially in all major earthquakes. In the Long Beach earthquake of March, 1933, a sand-spit peninsula more than 1 000 ft wide and several thousand feet long settled, and the land surface widened, causing cracks in paving aggregating as much as 5 ft. There has been serious settlement and longitudinal cracking in a large hydraulic-fill dam during an earthquake of such moderate proportions that practically no other damage to structures of any kind occurred. The reservoir was empty at the time. During the Santa Barbara earthquake of 1925, the Sheffield Reservoir was lost through complete failure of the small earth-fill dam which created it. Elsewhere, earth-fill dams have survived severe earthquakes.

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[16] Dean, School of Eng., Stanford Univ., Stanford University, Calif.

[16a] Received by the Secretary February 14, 1938.

During the Long Beach earthquake there was general settlement and consolidation of land surface in deep alluvial-filled valleys in the vicinity. The consolidation of alluvial materials resulted in a sudden rise of water in wells, and where ground-water was near the surface, of a large number of small jets of water flowing like little geysers, and leaving their record in fumarole-like deposits and erosions at the cracks in the ground surface through which the water flowed.

Of course, none of the physical conditions related to the foregoing observations is very closely related to the type of fill represented in properly constructed rock-fill dams. However, the settlement of mine and tunnel dumps has been observed as the result of earthquakes. Here, there may be a very marked resemblance to the normal loose rock-fill dam. The absence of placed rock, and a concrete blanket on one side of the fill, such as that used at the up-stream face of rock-fill dams, is not important in making such comparison. Of course, most of these mine or tunnel dumps will have too much fine material or will be too soft a grade of rock, and will not have included the important sluicing during rock placing. It is believed, however, that observation of such dumps may reveal some that closely resemble the loose fill of rock-fill dams. This subject is mentioned in the hope that engineers may be on the lookout for such rock-fills which may have settled during earthquakes, or which may be checked later if such earthquakes should occur in their vicinity.

With the hope that the author will comment further on the resistance of high rock-fill dams to earthquake shock the writer presents this discussion.

Mr. Galloway describes the cause and manner of settlement of rock-fill dams adequately, and yet briefly, in his first paragraph on settlement (see heading "Design: Settlement").

In discussing the cross-section (see heading, "Design: Cross-Section"; third paragraph) the author presents the accepted practice from which the typical rock-fill dam is shown to be composed of loose rock with a minimum top width of 15 ft and slopes equivalent to the angle of repose of from 1 on 1.3 to 1 on 1.4. For a thickness as great as 15 ft at the up-stream face, derrick-laid or packed rock is placed to form a semi-rigid layer to support the concrete or other impervious face.

In the case of a high dam, settlement of as much as 5% of the height may be expected owing to the spalling or crushing of point contacts between adjacent rocks in the loose rock-fill. As successive layers of rock are placed and the dam is built higher and higher these point contacts continue to spall for some time after completion of the structure until, finally, the points of contact between the rocks have sufficient area to carry the stresses involved in the dam standing without load.

These areas of contact are only sufficient to carry the normal mass of the dam itself. Many of the points of contact between the rocks will have just crushed sufficiently to carry this load at the point of incipient failure. The result is that when the water load is applied there is an immediate increase in unit stress, and the many small areas of contact already stressed to point of failure cannot carry the additional water load; renewed and further spalling takes place until the areas of these point contacts have increased sufficiently to be able to carry the forces due to the mass of the dam plus the water load on the up-stream face. This settlement may continue for months or years depending upon the rate of deterioration of the individual rocks composing the loose rock-fill.

Assume, now, that the reservoir is filled to its maximum high-water level and a severe earthquake occurs—not an earthquake involving any fault movement or permanent displacement of the rocks in the dam foundation, but merely the normal earth waves to be expected. These waves will give an oscillating motion to the foundations, exerting new and additional forces upon the rock mass. Point contact between rocks which have already spalled, first as a result of stresses due to the mass of the dam, and, second, to an additional extent, due to the water load, will immediately be stressed beyond the yield point. Additional spalling and crushing will result in immediate accumulated settlement.

This settlement may easily be much more extensive than may be computed from comparable known loading conditions. The vibrations and oscillations of the earthquake will likely tend to rotate the individual rocks in such a manner as to cause them to slide on their crushing points of contact, thus bringing about a denser compact mass of rock. Such immediate settlement and compaction during earthquakes have already been noted as a common phenomenon in all other types of fill, such as railway, highway, mine and tunnel dumps, and natural alluvial fills and sand-spits. Otherwise stable mountain slopes may yield to severe sliding, rolling, and sloughing of rock and rock masses. Such conditions were actually photographed on the slopes of San Jacinto Mountain during the San Jacinto-Hemet earthquake of 1918.

In other types of dams the vertical component of acceleration during earthquake vibrations is easily taken care of by the usual factors of safety in design. In the rock-fill dam even the vertical acceleration must be considered as it will add to the unit stresses in the point contacts, causing rock spalling and dam settlement.

The effect of sudden settlement of the rock mass upon the concrete lining on the up-stream face may easily be much more disastrous than the slow settlement that occurs during construction, during the filling of the reservoir, and thereafter. Reinforced concrete has generally been used for the highest rock-fill dams, and, therefore, reference to its use is made herein. Because of its high modulus of elasticity and lack of flexibility some failure of lining is likely to occur unless ample provision has been made in the design for such settlement.

The oscillations of the earthquake motion tend to free the rock at the face of the dam. If this rock is placed as steeply as the angle of repose there may be some likelihood of rocks being freed at the surface to roll, slide, or, possibly, to cause bulging.

The total settlement that may reasonably occur during an earthquake is difficult to foresee. Probably the settlement during construction, during reservoir filling, and for several years thereafter, will be some indication. That there will be such immediate settlement seems certain. It is quite conceivable that the settlement during earthquake shock may exceed all other settlement in magnitude. Some dams having too little free-board may settle sufficiently to cause over-topping. Fortunately, most rock-fill dams have been provided with large spillways and liberal free-board as it is well known that they cannot be over-topped with safety.

In a number of the rock-fill dams observed by the writer the irregular settlement of the up-stream face has been a substantial problem in maintaining water-tight conditions. The author suggests "that the fill should be composed of individual rocks of fairly uniform size, one rock bearing directly upon another" and that "any wide divergence in size will cause excessive and unequal settlements." Presumably, the important factor is that the size grading shall be uniform and of uniform quality of rock to avoid unequal settlement, rather than that all rock should be of the same size.

In constructing a rock-fill dam it is the general practice to sluice the fine material from the larger rock as it is placed. This fine material then tends to fill the interstices of the lower parts of the fill. As construction proceeds and the structure rises the lower part of the dam may become quite fully consolidated with fine materials in the interstices between the larger rock. The increasing load will then cause the points of contact of the larger rock carrying the load to spall off. If this process should continue sufficiently the resulting consolidation would cause an indeterminate part of the load to be carried on the fine material.

Under early normal static conditions such condition may not be entirely undesirable. However, with the passing of time, the smaller particles will become more disintegrated. Earthquake forces and vibrations may then tend to cause some greater tendency toward sliding either in whole or in part. Possibly this condition will be offset by more rapid consolidation of the small material than of the larger load-bearing rocks.

There are certain qualities inherent in high rock-fill dams which the writer believes should be considered in the design and construction of such dams. He has enumerated some of these in the hope that the author will discuss the resistance of high rock-fill dams to earthquake shock, and state the factors which in his study and experience should be provided in the design of such dams in regions where earthquakes are likely to occur.

L. F. HARZA,[17] M. AM. SOC. C. E. (by letter).[17a]—The rock-fill dam has proved its value in many places where transportation was difficult, concrete materials expensive, and other factors favorable. However, it should be pointed out that a rock-fill dam with concrete face will inevitably require some repair of the face after the initial period of settlement has passed.

This limits the proper application of this type of dam to storage reservoirs which are emptied at intervals in connection with their use, thus exposing the face for repairs. Where a large sacrifice in earnings would be entailed by emptying the reservoir, power dams are not a satisfactory application of the rock-fill type with concrete face.

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[17] Cons. Engr. and Pres., Harza Eng. Co., Chicago, Ill.

[17a] Received by the Secretary February 18, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

DESIGN OF REINFORCED CONCRETE IN TORSION

Discussion

By Messrs. Walter H. Wheeler, and A. Floris

Walter H. Wheeler,[16] M. Am. Soc. C. E. (by letter).[16a]—The principles set forth in Professor Andersen's paper have important significance when applied to the determination of correct moments of resistance in flat slab structures. Numerous tests on flat slab floors have shown a consistently wide disagreement between the stresses determined by the commonly accepted beam-strip theory of analysis and the actual stresses in the structure under load. Various authorities have explained the discrepancy on the theory that the tensile strength of the concrete was responsible, whereas it would appear from the paper that a more logical explanation would be the torsional resistance of the slab.

Since the torsional resistance is based upon shearing strength and not upon tensile strength of the concrete it would seem logical and proper to give credit to this factor in the flat slab formulas. It does not seem logical to base the analysis of the stresses in a continuous plate supported on columns upon the theory of independent beams supported on columns without any connection between those beams, as is done in the generally accepted flat slab theory.

The writer wishes to call attention to the results[17] obtained in the test of the Western Newspaper Union Building, in Chicago, Ill., in 1916, in which four adjacent panels were loaded. The panels were 17 ft 5.5 in. by 19 ft 4.5 in., with a nominal slab thickness of 8.5 in. The actual slab thickness at the edge of the column capitals was 8.35 in. The diameter of the column capitals was 4 ft 6 in. and the design load was 250 lb per sq ft. The test load over the full area of four panels was 913 lb per sq ft and the minimum recovery of the slab after loading was 75 per cent.

Note.—The paper by Paul Andersen, Assoc. M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1937, by Messrs. C. W. Deans, and L. E. Grinter; and January, 1938, by Messrs. Bruce G. Johnston, and Dean Peabody, Jr.

[16] Designing and Cons. Engr., Minneapolis, Minn.

[16a] Received by the Secretary December 21, 1937.

[17] *Bulletin 106*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

It is to be hoped that the torsion theory will be developed for flat slabs and that more accurate formulas will result. It has seemed to the writer that the 1917 report of the Special Committee of the Society on Concrete and Reinforced Concrete, on the design of flat slabs,[18] came much closer to an accurate design basis than the more recent formulas of the Joint Committee on Specifications for Concrete and Reinforced Concrete particularly as to slab thickness required where drop panels are not used.[19]

If the only aim is to be conservative without regard to accuracy, designers should perhaps be satisfied with the present thickness formulas. If they are seeking economical designs which are reasonably accurate, these thickness formulas should be modified.

The progress report of the Joint Committee on Specifications for Concrete and Reinforced Concrete for 1937,[19] gives a basis of determining slab thickness which is reasonably accurate and then recommends that having found this reasonably accurate thickness, 20% be added to it. The Committee proposes adding 20% to the thickness that would be determined as proper according to the 1917 report on the design of flat slabs.[18] Following the Joint Committee's recommendation, the slabs in the Western Newspaper Union Building would be increased from 8.5 in. to approximately 10 in. thick for 3 000-lb concrete.

By the application of the torsion theory to this slab, a reasonable and ready explanation is found for its remarkable performance, and the necessity for increasing the thickness is not apparent.

Further data on thickness of flat slabs with capitals and without drop panels are given in a published report[19] by the writer.

A. Floris,[20] Esq. (by letter).[20a]—From the academic point of view, the author's treatment of rigid frames, as systems in space, is valuable and instructive. In practice, however, there may be some question. The solution of the problem, even as applied to the simplest frame, is too complicated to be practicable. Consequently, the application of the method to frames as ordinarily designed will be out of the question, especially if the problem is complicated by side-sway.

If one considers that even a beam stressed by pure bending may be subjected to twisting moments by a beam connected to the same column, under a certain angle, it is evident that the proposed method is too theoretical to be practicable. For this reason, designers will continue to consider the analysis of building frames as a plane problem. In general, structural engineers should avoid direct torsion and, if this is not possible, they should at least minimize its effect by modifying the design.

The intent of this discussion is not to criticize the author's efforts, but to emphasize the need for keeping in mind the practical applicability of the theory. Professor Andersen deserves much credit in focusing the attention of engineers on the comparatively unexplored field of torsional resistance of rigid frames.

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[18] Final Rept., Special Committee on Concrete and Reinforced Concrete, *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 1135.

[19] Not yet published; see, also, "Comparison of Actual Flat Slab Designs with Code Requirements," *Engineering News-Record*, January 28, 1937, p. 132.

[20] Dipl.-Ing., Los Angeles, Calif.

[20a] Received by the Secretary January 24, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ECONOMICS OF THE OHIO RIVER IMPROVEMENT

Discussion

BY J. E. WILLOUGHBY AND W. D. FAUCETTE,
MEMBERS, AM. SOC. C. E.

J. E. WILLOUGHBY[29] AND W. D. FAUCETTE,[30] MEMBERS, AM. SOC. C. E. (by letter).[30a]—The problem offered for discussion in this paper is stated by the author as follows (see heading: "The Problem"):

> "Freight moves on the Ohio River because it can be transported more cheaply by water carriers than by land carriers. It can be transported at low rates because engineering works have been built on the river at the expense of the tax-payer. Since no tolls are charged, the Treasury derives no direct return. The public obtains an indirect return in the reduced net cost of water-transported articles consumed. The problem is to compare the engineering costs of the improvement with the value of the public benefits. * * *"

Assumption (1) is that the entire reduction of costs (the author means, not all the cost, but merely the boat operator's cost) in transportation of freight, due to the existence of an improved waterway on the Ohio River, is passed on to the public in the form of lower prices of the goods transported, and hence becomes a public saving.

His own discussion which follows indicates that this is a most violent assumption. In the report of the Mississippi Valley Committee of the Public Works Administration (PWA), submitted to Administrator Ickes on October 1, 1934, it was correctly stated: "Private carriers—largely coal and steel companies—now transport more than 95% of the total commerce on the Ohio River." Recently, there has developed a heavy volume of petroleum products moved in tank barges for the large oil companies.

As Fig. 1 shows, the really heavy tonnage of coal on the Ohio River is between Huntington, W. Va. (*via* the Kanawha River or by rail) and Cincinnati,

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NOTE.—The paper by C. L. Hall, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: December, 1937, by Eugene L. Grant, Assoc. M. Am. Soc. C. E.; and February, 1938, by Messrs. Fred Lavis, O. Slack Barrett, and Edmund L. Daley and Forrest E. Byrns.

[29] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

[30] Chf. Engr., S. A. L. Ry. System, Norfolk, Va.

[30a] Received by the Secretary January 24, 1938.

Ohio, where some of it is re-shipped by rail. The author quotes testimony indicating that any difference between the rail-river-rail and all-rail costs is not passed on to the consumer, but that the cost to the consumer is identically the same. The writers know of other evidence before the Interstate Commerce Commission indicating that all such coal is sold on the basis of the all-rail rates. The great bulk of the coal consumed along the Ohio River and contiguous territory is moved by all-rail routes, and it may safely be stated that in the great majority of instances any alleged "saving" is simply "pocketed" by the seller.

It is well known that iron and steel, moving in heavy volume the length of the Ohio River, are generally sold on the basis of certain mill prices (usually Pittsburgh) plus the rail rates from there to their destinations. The alleged "saving" is retained by the shipper or producer.

It is certain that the price of gasoline to the consumer is not less because of the heavy volume of water-borne gasoline on the Ohio River. It is a matter of common knowledge that these prices closely reflect the price in the Mid-Continent Field plus the cost (usually rail rate) of transporting it to its destination.

It is useless to consider sand and gravel, moving in heavy volume by barge, because the value of river improvements and maintenance to such traffic is nil, if not a detriment.

Assumption (2) is that the reduction of cost in transportation of freight is equal to the rail cost of transporting the freight actually carried by water, minus the water cost of transporting that same freight.

The author complains because rail costs cannot be determined, and it is necessary, therefore, to use rail rates; but water rates are as difficult to determine as rail costs and, therefore, it is necessary to compare rail rates with water costs. In doing so, however, it must be obvious that like is not compared with like. As will be developed later the author's alleged "water costs" are nothing more than general averages plus so-called "terminal differentials" and there are available general averages of rail charges in the form of average ton-mile earnings which would have permitted of like-for-like comparisons.

The author's definition of cost in Assumption (2) is subject to criticism. Assumption (3) requires no comment, and Assumption (4) is admittedly incorrect. It is possible to obtain a break-down of the expenditures of the Federal Lighthouse Service and of the Steamboat Inspection Service sufficient even to approximate the costs chargeable to the Ohio River. Whatever they are, large or small, they should be included to secure the true picture.

Assumption (5) is that, in comparing expenditures and benefits, no allowance has been made for taxes. This is correct; but why not? One reason assigned by the author is that such a charge is not applied generally to all carriers moving on public highways. Although it may be true that motor carriers do not pay adequately for the use of the public highways, they do pay something for such use in the form of special gasoline or mileage taxes, whereas the water carriers pay nothing. Another assigned reason is that nobody charges the cost of lighting an airway against an air line. This is an irrelevant statement when the purpose is to compare rail costs with water costs.

He then argues that the water carriers pay taxes on terminal properties which would have practically no value if the waterways were not improved.

Perhaps this is true, but he ignores the fact that such water terminal properties generally take the place of railroad terminal properties which are just as taxable. Then he states that they pay taxes on floating plants that would not exist if there were no waterways. Here, again, he ignores the fact that the floating plant takes out of commission, at the very least, an equivalent in railroad equipment which is taxable. The fact is that most of the water carriers incorporate in Delaware and thereby escape the payment of taxes anywhere on floating equipment. (In this connection see reports of the Mississippi Valley Barge Line Company and American Barge Line Company on file with the Interstate Commerce Commission.)

Furthermore, when the tax-free, improved, Ohio River waterway and the virtually tax-free barge companies operating over it engage in competition with the tax-paying railroads (paying in taxes approximately 7 cts of each gross revenue dollar), the result is not only to decrease the railroad volume and increase its unit costs, but it lessens the ability of the railroad to pay taxes needed for Government purposes, including public schools. The tax revenue loss thus created must of necessity be made up by heavier taxation on other property owners and taxpayers, as has been so well stated by Professor H. L. Lutz in "Public Finance," who stated further that the exemption of a utility from taxation means an exemption of the users of the service from taxation, to the extent of their use. In the present case, it means that a relatively few large steel, coal, and petroleum companies transporting 95% of the traffic on the Ohio River are receiving the benefit while the general taxpayers are paying the deficit. As bearing on taxes, the following quotation from Professor Lutz is apt:

> "There appear to be good reasons for the inclusion of a charge against current earnings roughly equivalent to the local tax on a similar private property. In the first place, such a charge is a proper element of cost which the industry should bear, whether it is publicly or privately operated. Secondly, the imposition of such a charge would throw into clearer relief the actual incidence of the benefits of public as compared with private operation. It is not desirable in a democratic community that any limited class should continue to receive benefits which are being paid for by the remainder of the group, unless this class is defective in intelligence and earning power, or is subject to handicaps of health and strength which require special acknowledgment. A third reason for the inclusion of such a tax charge is found in the fact that by such a practice some part of the cost of government is diffused among those who are not readily reached by means of property taxes or other direct forms of revenue."

It is true that taxes are not actually paid on the improved waterway and scant taxes are paid by the boat operators; but when, from an economic and social standpoint, an attempt is made to measure the relative economics of waterway *versus* rail transportation, an allowance for taxes must be made, either by adding them to the waterway costs or by deducting them from the rail costs. Simply to ignore taxes as is done by the author is untenable.

Referring to that part of the paper headed "Carrier Costs of Water-Borne Freight": The basis for the approximate costs, as stated, is somewhat obscure. There is no way for the writers to check any of the alleged "savings," in cents per ton, or in thousands of dollars, appearing in Table 3. For the purpose of analysis the writers will have to be content to use the "line-haul costs of 2 and 3 mills per ton-mile for principal coal movements, and about 3 mills per ton-mile for steel." Although the so-called "terminal differentials" generally used by the author are stated at the end of the paper, it is difficult to make use of them, so they are largely disregarded herein.

In discussing Table 4, the author admits the overwhelming importance of the four products, coal, steel, petroleum, and sand and gravel.

Under the heading, "Net Savings," the author states that the costs on each of the more commonly used routes for each important commodity were determined by multiplying the ton-mileage moved by the average cost per ton-mile as estimated (for water-borne freight), adding the terminal differential between rail and water transfers (except petroleum), and also adding the actual rate for rail haul where the commodity was hauled either to the river, from the river, or both; these costs were deducted from the comparative rail rates, and the gross savings established, and from this value the savings per ton-mile were obtained.

Solely for the purpose of discussion, the writers accept the Government costs, in mills per ton-mile (1934), of 5.6 as stated. This is a water ton-mile cost to the Government. It is conservative to state that, due to the meandering of the inland rivers, the water distance, except for railways that follow the meandering river, is on the average at least 50% longer than the short-rail distance. It is perhaps true that the average movement by rail may be as much as 10% (the former Federal Co-Ordinator computed it at 11%) longer than the shortest route. Consequently, to obtain a fair comparison of water-line costs (Government and boat operator) per ton-mile with rail revenues per ton-mile, the former must be inflated 50% and the latter 10%, thus bringing both to a short rail-mile basis.

In this connection, it should again be stated that sand and gravel, now and long before the present improvements, is and was dredged from near-by sand bars and moves short distances to large cities. By no stretch of the imagination can sand and gravel secure a material benefit, if any at all, from the costly river improvements and maintenance. It has been said, truthfully, that the pools formed by the forty-nine dams in the Ohio River submerge and otherwise destroy (by failure to create) sand-bars from which the sand and gravel is dredged; and yet, the cost of the river improvements and maintenance is spread over all traffic, including this heavy tonnage of sand and gravel. Table 4 shows that of the total of 13 018 335 000 ton-miles in 1934, 1 026 849 000 ton-miles were represented by sand and gravel. It is likewise doubtful whether the improvements and maintenance were of any value to the log and lumber traffic, the ton-miles of which in 1934 were 64 516 000. Sand, gravel, logs, and lumber represented more than 8% of the total ton-miles. If eliminated, and if the Government cost were allocated to the traffic that benefits therefrom, obviously, the average Government cost would be in excess of 5.6 mills per water ton-mile. For the purpose of comparison, however, 5.6 mills will be used.

Although it is stated that the average Ohio River haul is 119 miles, clearly this is depressed because of the influence of the short-haul sand and gravel traffic. In any event, it is not a very important factor in this analysis.

The rail carriers that are principally competitive with the traffic moving over the Ohio River waterway, together with ton-mile revenues in 1934, actual and as inflated 10% to make them fairly comparable with absolute short-rail hauls, are as shown in Table 8.

TABLE 8.—REVENUES FOR RAIL HAUL FOR 1934

Railroad	Revenues, in Mills per Ton-Mile	
	Actual	Inflated 10%
Chesapeake and Ohio	5.96	6.56
Norfolk and Western	6.62	7.28
Louisville and Nashville	8.18	9.00
Pennsylvania	9.15	10.06
Baltimore and Ohio	9.46	10.41

TABLE 9.—WATER TON-MILE COST

Operator	Rate, in Mills per Ton-Mile	
	Actual water ton-mile	Equivalent rail ton-mile
Government	5.6	8.4
Water operator	2.0	3.0
Total (water cost, 2¢)	7.6	11.4
Government	5.6	8.4
Water operator	3.0	4.5
Total (water cost, 3¢)	8.6	12.9

Each of these carriers transports a heavy volume of the same bulk commodities as are transported on the Ohio River, principally coal. Out of these earnings they must pay all costs, including taxes. With the exception of the Baltimore and Ohio Railroad all were prosperous, or fairly so. No one can deny that for comparative purposes they represent a fair selection to determine the relative true costs of moving traffic on the Ohio River compared with railroad haulage.

Ignoring entirely any factor of excess terminal costs of waterway over rail, and the exemption of water transportation from taxes, and merely using the average of 5.6 mills per water ton-mile, Government cost (also as inflated 50% to reflect an equivalent rail ton-mile) and water operator costs of both 2 and 3 cts (see heading, "Carrier Costs of Water-Borne Freight") per water ton-mile (also as inflated), the values shown in Table 9 result.

It will be observed that these partial waterway costs are greatly in excess of any of the foregoing rail lines. If average ton-mile costs are to be used for water transportation then it is eminently fair to use average ton-mile revenues for the railways; in fact, their use is decidedly favorable to the waterway transportation.

Fig. 1 shows a heavy tonnage of coal between Huntington and Cincinnati. This coal originates on the Chesapeake and Ohio Railway which line has established very low rates to Huntington for the movement beyond that point *via* river; it is unloaded through a tipple into barges at Huntington. Most of the coal then moves to a point just west of Cincinnati where it is unloaded, moved up an incline through a tipple and dumped into rail cars for movement to points in and about Cincinnati and points beyond. It is obvious that the barge movement between Huntington and Cincinnati is simply a substitute for Chesapeake and Ohio rail service between the same points. The Chesapeake and Ohio runs parallel to the Ohio River between these points, and both the river and the railroad are about 160 miles long. Movement *via* barge necessitates two intermediate terminal services which are avoided on all-rail movements.

Computing the barge movement at 2 mills per ton-mile, a low estimate (see author's discussion of Table 3), and loading at Huntington at 5 cts per ton, unloading at Cincinnati at 15 cts per ton (see "Terminal Differentials"), plus the Government cost of 5.6 mills per ton-mile, the total cost for 160 miles is $1.42 per ton.

The average ton-mile revenue of the Chesapeake and Ohio Railroad in 1934 was 5.96 mills. This includes all terminal expenses. This railroad had an average haul in 1934 of 284 miles. Although the substitute haul, in this case, is less, no additional origin terminal expense would have been necessary if handled through by the road, and the same would have been true if the coal had been consigned to points beyond Cincinnati. The average of 5.96 is conservative. It produces 95 cts per ton for the rail haul from Huntington to Cincinnati, as contrasted with a total cost of $1.42 per ton for the substitute river service. Manifestly, a movement *via* river is profitable to the transporter only because the taxpayers through the Federal Government contribute 90 cts of the true cost of such a movement.

It is probable that the alleged savings as determined by the author (Table 4) would not stand the test if it were possible to check his computations. In this connection note Table 6 and the author's discussion of Table 6. What he does here is to assume movements from Cincinnati proper to Louisville, Ky., proper (note this carefully) of:

Freight	Movement, in thousands of tons	Equivalent movement, in millions of ton-miles
Coal and coke	10 805	741
Cement	108	21
Iron and steel	844	473
Oil and gasoline	1 272	201
Unclassified	939	288

He then proceeds to determine the rail costs to a shipper per water ton-mile by using the local rail rates from Cincinnati to Louisville as contrasted with an assumed water cost per water ton-mile. A local rail movement of coal from Cincinnati to Louisville is almost impossible. No coal originates there and the rail rate used is virtually meaningless. The fictitious rail movement used represents nearly 50% of the total ton-miles used in the comparison, and greatly affects the showing. There is little likelihood of any appreciable movement of cement or iron and steel either by rail or by water locally, from Cincinnati to Louisville. The entire comparison partakes of the nature of aligning data in a direction favorable to the project and to produce a desired result.

By the method used, the author is able to produce an average rail cost to the shipper on the aforementioned traffic from Cincinnati to Louisville of 21.3 mills per water ton-mile as contrasted with a water cost of 14.9 mills per water ton-mile (cost to shipper 9.3 mills plus cost to Government 5.6 mills), and yet the average ton-mile earnings of the Louisville and Nashville Railroad Company and the Baltimore and Ohio Railroad Company (also operating between Cincinnati and Louisville) were only 8.18 and 9.46 mills, respectively.

Under the heading, "Non-Statistical Considerations," the author apparently attaches much value "to the regulating effect of an improved waterway on the rates charged by other carriers." This can only be predicated on an Interstate Commerce Commission so impotent as to permit rail carriers to maintain rates higher than are just and reasonable. Although it is true that, in many cases, water competition has depressed the rates of the rail lines, it must not be overlooked that: An efficient railway system serves the entire country; it serves points on the rivers as well as in the interior; it must have operating revenues sufficient to pay all expenses, taxes, and a reasonable return on investment; the people as a whole must of necessity support the railway plant by paying rates that will produce sufficient revenues in the aggregate; and that were it not for the necessity of reducing some rail rates to meet subsidized water competition which benefits a relatively small part of the general public, the transportation burden might be more equitably distributed. The writers cannot agree that the so-called "regulating effect of an improved waterway" furnishes any economic justification for the huge expenditures of the taxpayers on waterways. No Government function, on one hand, should assume that another Government agency is not alive to its duties.

If the author had chosen to balance alleged benefits of this kind against disadvantages, he might have mentioned the facts that: The 49 dams have created a series of 49 pools, or lakes, averaging 20 miles in length; in periods of slack-water the current is almost negligible; the sewage dumped in the river at the numerous large cities has made the Ohio almost an open sewer, with a stench so bad in summer as to be noticeable, and even chemicals cannot destroy the bad taste of the city water at such periods; and, due to these virtually stagnant pools, all the large cities along the Ohio River must eventually install sewage disposal plants at heavy cost.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

SOLUTION OF TRANSMISSION PROBLEMS OF A WATER SYSTEM

Discussion

BY MESSRS. WARREN E. WILSON, AND HAROLD K. PALMER

WARREN E. WILSON,[18] JUN. AM. SOC. C. E. (by letter).[18a]—An interesting application of graphical methods to the solution of problems encountered in the design of water distribution systems, is presented in this paper. It is claimed for the method that in certain cases better progress can be made than with mathematical methods or by trial and error. It is not entirely clear which cases are in this category. Since the purpose of the graphical solution is avowedly to obtain a practical result in a short time it should be on the basis of this purpose that it be compared with other methods. Great accuracy is not required; nor can it be expected, since the basic assumptions one makes are necessarily merely approximations. Hence, any of the methods may be expected to give results that have the required degree of accuracy.

It appears that the preliminary process through which one must go, namely, that of plotting curves to suitable scales, necessarily consumes considerable time. In addition, one must adjust a number of curves to the proper positions to obtain the desired results. This adjustment is doubtless made with greater speed as one gains experience in the method, but must nevertheless take considerable time. The fact that considerable experience is required to use the graphical method to advantage then appears to be an undesirable feature of the method since it is possible to teach the Cross method to a group of students and develop in them sufficient skill to handle complex networks in a period of not more than a week. It would be difficult to develop the same proficiency in the graphical method in a similar period of time. From the standpoint of the professional engineer the need for an experienced designer in the manipulation of such a method is a disadvantage since the work could be delegated to an inexperienced person after giving him just sufficient instruction to enable him to handle the method of successive approximations as a routine set of calculations.

NOTE.—The paper by Ellwood H. Aldrich, M. Am. Soc. C. E., was published in October, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1938, by Messrs. Lynn Perry, Charles M. Mower, Jr., and Thomas R. Camp; and February, 1938, by Messrs. Harold E. Babbitt, and C. Maxwell Stanley.

[18] Asst. Prof. of Civ. Eng., South Dakota State School of Mines, Rapid City, S. Dak.

[18a] Received by the Secretary January 31, 1938.

It is the writer's belief that if one acquaints himself fully with the fundamentals of the Hardy Cross method[4] and clears away any false notions he may have about a method of successive approximations in the question of accuracy, he can solve even the most complicated network problems in a comparatively short time. There can be no question that the preliminary preparation required in the Cross method is far less than that required for the graphical method, since it involves only the calculation of a constant for each section of pipe in the network. The actual work that is necessary in making the successive approximations is certainly no more difficult and requires very little time.

The writer would be interested in a statement of the particular types of problems in which the graphical method is believed to be superior to the Cross method.

HAROLD K. PALMER,[19] M. AM. SOC. C. E. (by letter).[19a]—The method of handling complex problems in water supply design, which is presented in this paper, well merits the careful study of all who have similar problems to solve, and the author deserves praise for the ingenious manner in which he has solved some of the worst of them. The writer has used the same method in handling pumping problems, which the author mentions, for several years and has found it a very convenient tool. The following comments are offered to supplement the author's handling of the subject and to emphasize the fact that it is a method which can be used for many purposes, depending largely upon the ingenuity of the engineer.

One fact that should be especially emphasized is that there is nothing obscure or complex about it. Instead of trying to solve algebraic formulas of the second or higher degree by computation the designer simply plots the curves, combines these curves, and the intersections give the solution desired. Thus, the solution of the problem is no more difficult than the drawing of the curves. It is only necessary to keep in mind the factors that produce loss of pressure and to apply them in the right places by simple methods. The principal way in which it differs from the ordinary computation is that the latter is performed for one pressure or one value of Q, and if the result is not correct another approximation must be made. In the author's method all values of pressure and Q within certain limits are considered and the solution is given at once by the intersection of certain lines.

Fig. 1 was prepared probably from the Williams-Hazen tables, which is also the writer's method of making similar drawings, but these tables give only standard cast-iron pipe sizes, and other sizes must be interpolated. This can be done very easily by the following method, which is applicable to any similar exponential formula. Fig. 15 is a simplification of Fig. 1 showing only a few of the lines to avoid confusion, and yet show the method. Marking the intersections of the line for 20-in. pipe with the 20 ordinate, the 30-in. pipe line with the 30 ordinate and the 60-in. pipe line with the 60 ordinate, it will be noted that all the intersections marked are on the same straight line, which can be called the interpolating line. Why this should be a straight line can be proved by any one mathematically inclined, but it is not required in this case, and it is easier to draw lines for two or three sizes of pipe and join the proper intersections than to compute the slope of the intersecting line. To interpolate any other size of pipe it is necessary only to note the intersection of the interpolating line with an ordinate corresponding to the diameter required, and draw through this intersection a line parallel to the other pipe curves. Thus, any size of pipe can be interpolated without difficulty and in much less time than it takes to find the values in a table and plot them. Furthermore, a graph for metric sizes could be computed from the data for English sizes.

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[4] *Bulletin No. 286*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

[19] Chf. Draftsman, Los Angeles County Sanitation Dists., Los Angeles, Calif.

[19a] Received by the Secretary January 31, 1938.

Several manufacturers of centrifugal pumps are using a method similar to that of the author to show the behavior of pumps on a given pipe system. In this case, it is more convenient to consider the system curve for the pipe as a static lift, plus friction, giving a curve in the upper right-hand quadrant which is concave upward. The ordinary head-capacity curve for the pump is then superimposed on it. This latter is concave downward so the two intersect at a sharp angle and show at once how much water the pump will handle. Should it be necessary to start a second pump its curve is plotted in the same quadrant and the two are combined by adding the respective abscissas for common ordinates, and the intersection of this curve with the system curve shows the quantity the two will handle, which, incidentally, is less than the sum of the two individual capacities when running alone. In Fig. 16, *SS* is the system curve; *AA* the curve for Pump No. 1; and *BB* for Pump No. 2. Points *a* and *b* show what each pump will do alone; *CC* is the curve for both pumps; and Point *c* gives the quantity of flow that the two will handle.

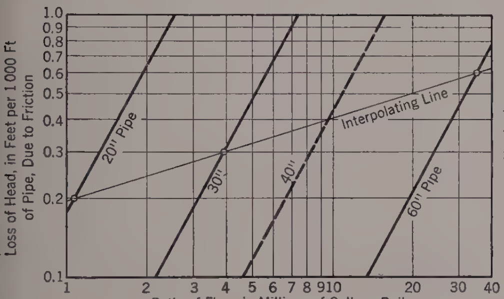


FIG. 15

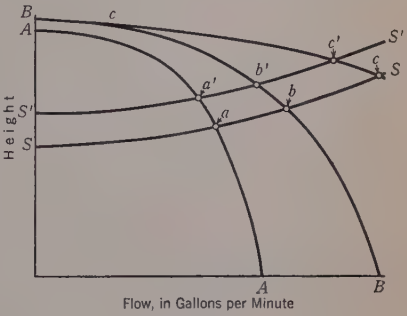


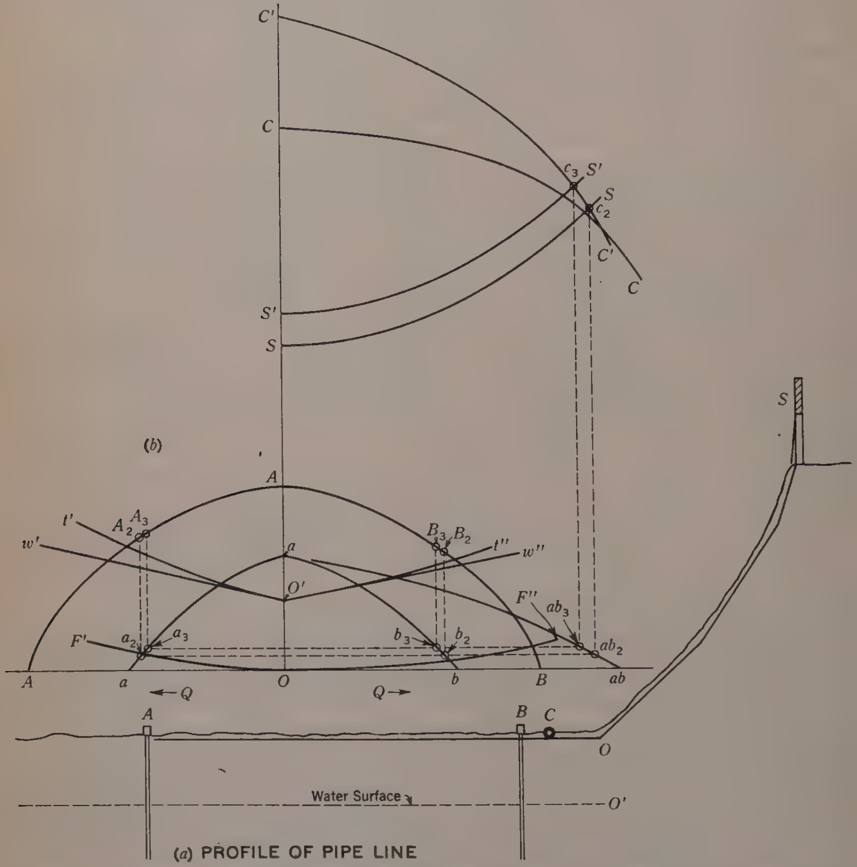
FIG. 16

If the pumps are discharging into the bottom of a tall stand-pipe in which there is considerable fluctuation in the height to be pumped, it is advisable to draw another curve for the stand-pipe full, such as Curve *S′S′*, Fig. 16, in which the height, *SS′*, is the increased depth. Points *a′*, *b′*, and *c′* now show what the pumps will do as the stand-pipe fills.

Should the system consist of two or more pipes in parallel they can be added to the diagram in a similar manner and the saving in head, and increase in output of the pumps, is shown at once.

The foregoing method serves very well where the pumps are close together and the individual suction losses are small in comparison with the discharge head. It is obvious that if two 1 000-gpm pumps are running in parallel the friction back of the junction of the two is no more for both pumps running than it is for only one. In fact, it is rather less because each alone might pump 1 100 gpm and only 1 000 gpm when both are running.

For a comprehensive case consider two deep-well pumps, A and B, Fig. 17(a), separated several hundred feet, both feeding into the suction of a booster pump, C, near Pump B, which forces the water into a high stand-pipe far enough away to offer some appreciable pipe friction, and subject to an increase in depth with its equivalent increased pumping lift; also let the standing water surface in the two wells be at the same elevation but with different draw-down characteristics for each. To avoid too many overlapping lines in Fig. 17(b) the two well pumps are shown in different quadrants. The distance from the static water surface to the suction of the booster pump, C, is OO'. As water in Well A draws down, this lift increases along the line, $O'w'$. The suction losses and pipe friction to Pump C for Pump A are shown by Curve Of', and the total



(a) PROFILE OF PIPE LINE

FIG. 17.—LOSS OF HEAD, IN FEET PER THOUSAND FEET OF PIPE, DUE TO FRICTION

lift and losses for Pump *A* are shown by $O't'$. Arc *AA* is the head-capacity curve for this pump, and subtracting (vertically) $O't'$ from Curve *AA* gives Curve *aa* which represents the elevation at which Pump *A* will deliver various quantities of water to Pump *C*. A similar system is shown for Pump *B* in the right-hand quadrant, *ab* representing the elevation at which Pump *B* delivers water to Pump *C*. Adding *aa* and *ab* horizontally gives Curve *a–ab* which is the corresponding curve for the two running in parallel. Curves *SS* and $S'S'$ represent the system curves for the stand-pipe empty and full, respectively, and *CC* is the head-capacity curve for Pump *C*. Adding (vertically, since the two are now in series) Curves *a–ab* and *CC* give Curve $C'C'$, the combination curve for all pumps, intersecting the system curves at Points c_2 and c_3, which represent the quantities pumped when the stand-pipe is empty and full. Lines drawn vertically from these points to Curve *a–ab*, and then horizontally to Curves *ab* and *aa* with vertical lines from these intersections to Curves *AA* and *AB*, show at Points A_2 and A_3, and at Points B_2 and B_3 how much each well pump is handling, and if it is working at its best efficiency.

The theory on which this method of attack is based is that the pump adds pressure to the water, which pressure is reduced at any point along the pipe by the static lift plus the friction. These respective losses are charged against each pump at the junction point where it is obvious that both must be exerting the same net pressure, and the combined flow will be the sum of the flows corresponding to this net pressure. Since there was only one booster pump and no complications it was not necessary to use the same method on it, although it could be done if so desired.

This method of analysis is very useful in investigating low-head complex pumping systems and easily solves some very difficult problems with remarkable accuracy.

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DISCUSSIONS

MULTIPLE-STAGE SEWAGE SLUDGE DIGESTION

Discussion

BY MESSRS. WILLEM RUDOLFS AND H. HEUKELEKIAN, AND HERMANN BACH

WILLEM RUDOLFS,[18] M. AM. SOC. C. E., AND H. HEUKELEKIAN,[19] ESQ. (by letter).[19a]—The attempt to make use of the observed phenomenon of automatic segregation of solids during digestion by withdrawing particles of digested sludge from the bottom and allowing the more active mass to remain longer in the tanks is of unusual interest. It is well known that a portion of properly seeded fresh solids is rapidly decomposed, another portion will sink to the bottom of a tank, and still another part of the fresh solids (including fatty material, hair, match sticks, soap, etc.) remains floating for a considerable time while undergoing decomposition. Withdrawal of the material segregated at the bottom causes a gradual increase in the quantity, and frequently in the density, of the floating mass. This inherent behavior of the sludge is well illustrated by the authors in Fig. 4. Inability of the scum to move with the separated sludge to the next tank results in scum formation and this material may stay in the primary, secondary, or even tertiary tank for a long time, while the inorganic and partly digested solids pass on. This method of segregation is undoubtedly theoretically correct, because it permits the material most difficult to digest to remain longest in the tanks. On the other hand, the method results automatically in increasing quantities of scum in the first tanks, unless the digestion time allowed is ample to take care of the accumulating mass, or distribution of sludge; that is, unless so-called sludge circulation is practiced. If the digestion time is insufficient the mass of scum eventually

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NOTE.—The paper by A. M. Rawn, M. Am. Soc. C. E., A. Perry Banta, Assoc. M. Am. Soc. C. E., and Richard Pomeroy, Esq., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 16, 1936, and published in November, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1938, by Edward W. Moore, Esq.

[18] Prof. and Head of Depts., Water Supplies and Sewage Disposal, Rutgers Univ.; and Chf., Div. of Water and Sewage Research, State Agri. Experiment Station, New Brunswick, N. J.

[19] Associate, Div., Water and Sewage Research, State Agri. Experiment Station, New Brunswick, N. J.

[19a] Received by the Secretary February 15, 1938.

becomes so large that the capacity of the tanks is reduced materially and the scum must be taken care of by other means. The results graphically shown in Fig. 4 seem to indicate that after twenty-five months of operation the capacities of Tanks 1, 2, and 3 had been reduced (on account of scum formation) by 25 to 30 per cent. The tanks were emptied and cleaned, and it would be of interest to know what was done with this scum, which apparently digested actively when placed in suitable containers.

Of considerable interest is the fact that the average solids content of the new sludge pumped to the digesters was 4.7 per cent. The sludge mixtures consisted of 33% of primary and 67% of activated sludge by volume. This rather high solids concentration may be a contributory factor for the short digestion period in the digesters. In considering the short digestion period of 10.3 to 13.5 days, the open pits, serving as balancing reservoir, storage tank, and secondary digestion unit, are of importance. The scum formation, occasional foaming, and density of the subnatant and supernatant liquors indicate that the digesters were pushed to the limit and occasionally, possibly, beyond the limit. The foaming probably caused by the combination of using occasional poorly digested sludge for seeding and rapid gasification in some of the units, is perhaps the best indication of over-loading, together with the fact that supernatant liquor removal was promoted in the pits after storage.

The fact that good digestion results were obtained during the short digestion period in the tanks, followed by after-digestion in the pits, is of practical interest, but is not surprising. Similar results have been obtained elsewhere in the laboratory, in pilot plant trials, and in full-scale plant operation. Where no after-digestion facilities were available, sooner or later the difficulty was to produce persistently a rapidly draining, digested sludge. In a large part of the country open-air drying of sludge, which retains its moisture tenaciously, is limited to summer or to warm, dry seasons. The importance of secondary digestion should not be over-looked and, admittedly, it is a factor in the case under discussion. The over-all results obtained in the tanks, a reduction of 46 to 47% of volatile matter, indicate satisfactory digestion. This fact, together with the separation of water in the storage pits, seems to leave no doubt that the process utilized was successful as far as volatile matter reduction and gasification were concerned.

Actively digesting masses of sludge do not show temperature stratifications. The mass is in constant movement and, as has been stated previously, fresh and actively digesting solids entrain gases more readily than old or digested solids. The proportion of solids carried up by entrained gas depends, therefore, on the relative quantities of fresh solids. In an actively digesting mass of sludge the solid particles are in constant motion, either up or down, depending upon whether or not they carry entrained gas, so that the liquid and its contents are in a state of dynamic equilibrium. Since the sludge masses in the tanks were constantly stirred by the gas formation and the flow of sludge from tank to tank, only little, if any, temperature stratification would be expected. In cases where sludge is digested and stored in a single tank, the picture is different, depending upon the mass of actively digesting material and the capacity of the tank. When the storage capacity of the tank is large and considerable quantities of ripe sludge are retained, temperature stratification is likely to be encountered.

The data submitted on heat transfer, although limited, are of interest. The difference of 500 000 Btu between the heat applied and the heat required to raise the temperature of the new sludge was only 3.6% of the total heat input. Without having detailed results it is difficult to state how much of the loss can be accounted for by experimental error. Unfortunately, no results are available for possible radiation losses. The information on this phase of the problem is scarce, but seems to indicate that the difference in free-heat content of the raw and digested sludge and its subtraction from the total heat input, could be accounted for by radiation losses.

The general statement of the authors, that the 25% of sludge wasted directly from the digestion tanks to the drying beds showed no difference in drying characteristics from the 75% of sludge pumped from the pits, does not show what the cause was of the "rather erratic results, indicating considerable variation in drainability of sludge from time to time." In general, the 4.5% sludge lost about one-half its moisture in 48 hr. Extensive investigations with partly and well-digested sludges have shown that the quantity of water drainable on sand beds during the first 24 hr varies greatly with the degree of digestion. Well-digested sludge (52% solids) loses as much as 85% of its total moisture (at 74° F) by drainage, and the remainder is lost by evaporation. With increasing temperatures, evaporation increases. The drainage from incompletely digested solids is much less and slower. Even when 80% of the possible gas production has evolved, drainability is comparatively slow, amounting to from 40 to 50% of the total moisture content in 24 hr.

Nitrogen balances made some years ago[20] showed that there was no reduction in the total nitrogen content during digestion of sewage solids. On the basis of original solids the percentage of nitrogen increases in the digested sludge on account of the decomposition and loss of other materials into gases. The authors reach the conclusion that "the digestion of nitrogenous matter roughly parallels the digestion of other organic matter, with the result that the percentage nitrogen in the solid organic matter does not show any great change." This is similar to the conclusion reached previously. As the authors state, this does not mean that the nitrogen in the raw sludge does not undergo transformation, converting the insoluble nitrogen into soluble forms. During digestion, active hydrolysis of nitrogenous materials in the sludge takes place, resulting in an increase in ammonia, amino acids, and other soluble forms of nitrogen. The possibility of liberation of nitrogen gas from organic matter under anaerobic conditions does not seem great. Nitrates, if present, would be reduced, under anaerobic conditions, with the liberation of nitrogen gas. Nitrogen may also be formed by the interaction of nitrites with amino acids, but under anaerobic conditions neither nitrates nor nitrites could be produced. The loss of nitrogen from digesting tanks really occurs by the discharge of supernatant liquor, containing the various soluble nitrogen compounds. A second, and material, loss occurs when the digested sludge is placed on sand drying beds. The soluble nitrogenous materials disappear with the water drained. If these soluble nitrogenous materials could be retained, the digested sludge would have considerable value as a fertilizer. Activated sludge contains more nitrogen because a large part of the soluble nitrogen is transformed into protoplasmic nitrogenous substances, but the loss of total organic matter is less. When the soluble nitrogenous substances are retained in digested sludge, the nitrogen content should not only be higher on the basis of dry solids, but the nitrogen would be more available for plant food. At present, processes are known and in operation by which at least a part of the soluble nitrogen can be held in the digested sludge.

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[20] *Industrial Engineering Chemistry*, Vol. 20, p. 177 (1928).

The outstanding contribution of the work presented by the authors is that with proper operation an apparent digestion schedule of fifteen days can be maintained at a temperature of 85° F, utilizing a system of multi-stage digestion for the gradual and partial destruction of organic solids, producing the usual quantities of gas, leaving a drainable residue, without undue difficulties in operation.

Short digestion periods are of practical interest and may be obtained with modifications in environmental conditions. With the present state of knowledge the most important environmental factor for properly seeded sludge is temperature. When the temperature is raised high enough to obtain thermophilic digestion, the time required is materially reduced, but the principal disadvantage is that the sludge digested in a period of a few days does not de-water as rapidly as well-digested mesophyllic sludge. Results obtained on a system consisting of consecutive digestion at high temperature (130° F) for a few days, the digestion at low temperatures (80 to 85° F) for the destruction of remaining finely divided material and compacting of sludge, indicate that this is a practical and economical plan. Such a method must include adequate stirring mechanisms to obtain consistently good results. In a system of multi-digestion units, the basic principles of decomposition are not different from single-stage, two-stage, or consecutive digestion. It becomes mainly a question of economics of construction and operation, therefore, to determine which system should be chosen. The fact that the authors have shown that short-time digestion periods can be obtained with a multi-stage digestion unit, which does not require excessively careful operation, is of practical importance to those who must select a system. The greater the variety of methods, the wider the scope.

HERMANN BACH,[21] ESQ. (by letter).[21a]—The authors have rendered valuable work in contributing to the bio-chemical and physico-chemical reactions involved in the digestion of sewage sludge. The idea of stages, especially, is basically appealing as it is obvious that the more complicated a process is the more easily it may be simplified by sub-dividing it into convenient working stages, planned so as to favor the particular factor or factors segregated in each stage. In planning such stages, the designer should avoid the temptation of sub-dividing the process into more parts than may be justified economically, in order that the advantages of the separate stages may not be over-balanced or canceled by excess construction costs or unreasonably high operating cost. This kind of foresight is amply evident in the authors' description of the treatment plant constructed for the Los Angeles County Sanitation District. Although two-stage sludge digestion plants using a separate tank construction for each stage are in common use, the authors' description of a four-stage digestion system is remarkable not only because of the multiple stages, but also because, in solving the construction problem, the designers have avoided separate digesters for each stage, by confining the four stages to one structure. This device cleverly avoids the danger of excess first cost and increased operating cost.

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[21] Cons. Chemist, Berlin, Germany.

[21a] Received by the Secretary January 27, 1938.

The paper raises a logical question as to the number of stages that may actually be necessary in a given case. In the present case, why were there four instead of three, or five, or eight stages? It seems impossible, theoretically, to predict the number of stages that would be most advantageous and, therefore, one must be content to trust his judgment from the results obtained after the plant has been in operation. Selecting gas production as a "yardstick," the data presented in Table 5 afford an opportunity to determine the extent to which the Los Angeles plant has fulfilled expectations. Placing the total gas production (see last column of Table 17) as 100% of the "technical digestion range" (that is, omitting from consideration, the potential surplus of gas production listed in Table 6), the percentage of gas production by stages will be as shown in Table 17. Thus, the proportion of gas production (that is, the digestion work output in Stages 1 to 4) appears to be 1 : 0.46 : 0.40 : 0.36. Since the capacities of the four compartments are equal, whereas Table 17 indicates that the output is not equal (it is less in each stage), the base sub-division in the over-all tank apparently is not consonant with the relative efficiency of the four stages. Whereas, the sub-divisions were made equal in capacity, Table 17 indicates that, relatively, they should have been in the proportion of 1 : 2.1 : 2.48 : 2.8, with the capacity of the first stage assumed equal to 1. In the reverse order, with the fourth stage assumed equal to 1, the relative sub-divisions should have been 0.36 : 0.75 : 0.89 : 1. The dimension of each compartment in that case would have been proportional to the efficiency of the separate stages, thus fully justifying the construction cost.

TABLE 17.—AVERAGE PRODUCTION OF GAS, LOS ANGELES COUNTY SANITATION DISTRICTS, CALIFORNIA

Units	Stage 1	Stage 2	Stage 3	Stage 4*	Total, Stages 1 to 4
Thousands of cubic feet per day	125.3	59.3	50.4	45.4	280.0
Percentage of total	45	21	16	18	100

* The tanks in Stage 4 were only partly filled; the values given in Table 5 were computed as for full tanks.

The need for increasing the dimensions of the compartment in a multiple digestion plant is due to the fact that, in the first period, the sludge decomposes very quickly. As shown in Table 5, after the new sludge in Stage 1 has been substantially decomposed, there is a sudden decrease in the digestion rate, after which subsequent digestion is at a comparatively slow rate tapering to a point beyond which it ceases to be within the "technical digestion range" (a term proposed by Dr. Karl Imhoff). As far as possible, these peculiarities of the material dealt with should be given appropriate consideration in designing the structures for multiple-stage digestion.

The difference in gas production between Stages 3 and 4 is so slight as to indicate that the rate of decomposition had become uniform in each of them. Consequently, it may be unnecessary to construct more than three compartments if the third compartment is made proportionately larger than the first two. On the other hand, however, the more stages that are provided, the greater the opportunity will be for the plant to profit by the "gravimetric segregation," which is an exceedingly valuable feature of the process described in the paper. Indeed, this action may be, to a large extent, the explanation for the astonishingly short time period required to prepare the sludge for the drying bed.

Table 6 indicates that the sludge leaving the last digestion compartment is still capable of producing about 80% more gas if the digestion is extended to approximately twice that of the preceding four stages. It is to be regretted that this volume of gas is lost, which also raises the question as to whether the sludge has actually been digested sufficiently if it still contains some organic substance capable of such gas production. On the other hand, gas production is not the principal purpose of sludge digestion, the gas being only a by-product. The principal purpose, really, is to convert the sludge into a form that can be readily disposed of without creating a nuisance. As this objective seems to have been attained satisfactorily in a brief time at the Los Angeles plant, the designers were probably not justified in providing for an extension of the process merely to produce gas. Furthermore, there would be the potential danger that prolonged digestion would make the sludge too thick to handle in the pumps and on the sludge bed.

Scum Accumulation.—Scum is formed by fat particles which, because of their low specific weight, rise to the surface, carrying other particles with them. The percentage of fat and oil in raw sewage depends on local conditions such as the characteristic food consumed, the presence of industrial plants, automotive traffic, etc. The percentage of fatty material may vary greatly from plant to plant and within the same plant, from season to season. The intensity of scum formation in the sludge digestion tanks may vary accordingly. The so-called mineral oils, such as lubricating oil and grease, gasoline, etc., are highly resistant to bacterial decomposition. Nevertheless, the various fats of animal and plant origin, including the fat contained in greases, are very susceptible to anærobic decomposition, being an important source of sludge gas production as shown in Table 12. To produce and maintain such an anaerobic bacterial action, two conditions are necessary: Sufficient moisture and isolation from the atmosphere. In digestion tanks of the usual construction, including the Los Angeles plant described in this paper, these two conditions are not fulfilled satisfactorily. As may be noted in Table 3, the scum is too dry, containing less than 85% moisture. Such relatively small water content makes the unrestrained action of bacteria more or less difficult. In addition, the scum is freely exposed to the "air." Although, in this particular case, the "air" is not the freely circulating atmosphere, it is sludge gas, which is highly unfavorable to bacterial action because it is made up of products of bacterial metabolism.

In order to accelerate the digestion of fatty substances which form scum layers, means should be provided for moistening the scum throughout its total mass, and preventing its contact with circulating atmosphere or with sludge gas. An improvement in the process of draining the sludge could be devised by placing an elevated sludge tank between the digester or the sludge pit and the sand bed, thus making it possible to discharge the sludge on the bed by gravity. Pumping releases the gas from the sludge, thus raising its specific gravity. The result is that the particles drop quickly to the sand, clogging it against the supernatant. This unfavorable condition may be remedied by retaining the pumped sludge in the elevated tank for an additional digestion period of a half day or so. When the sludge has regained some gas content and is then discharged on the sand bed, the sludge will drain better as its particles will show a tendency to remain on top while the free water below flows through the filter.

The writer was especially interested in Table 9 which contains some very instructive data, the form in which phosphorous occurred is peculiar indeed. Some years ago, the writer observed a surprisingly high phosphate content in the scale deposited on heating coils in a separate sludge digestion tank operated as a whole at mesophilic temperatures. In investigating this strange case, he concluded that the thermophilic bacteria thriving in the immediate vicinity of the heated coils absorb phosphorous compounds to a higher degree than do the mesophilics in the remaining part of the digestion tank. After the thermophilic bacteria on the surfaces of the coils had died, there remained a residue accumulating slowly and being converted to mineral phosphates. It would be interesting to know whether a similar condition occurs in the scale deposited on the heating coils of the plant described by the authors.

Corrections for *Transactions:* On page 1674, following sixteenth line from the bottom, insert the paragraph: "Referring to Fig. 2, manways, gas collection, re-circulation and heating lines, sampling and thermometer wells are common to each tank; lifts are common to Tanks Nos. 1, 2, and 3; and raw and seed sludge lines go to Tank No. 1 only"; change the title of Table 2 to read "Composition of Sludge by Stages"; on page 1679, Line 9, delete "and Table 2"; in Table 6, Column (2), change the heading to read "Detention, in days"; on page 1684, Line 12, change "head" to "heat"; in Table 9, Column (1), transpose "sulfate, SO_4" and "Phosphate, PO_4"; on page 1688, Line 9, change "September" to "October"; in Table 12, Column (1), change "Phosphine (PH_3)" to "Phosphate (P_2O_5)"; in Table 14, the heading for Columns (4) and (5) is "Ether Soluble Matter"; and, in Table 15 (*b*), Column (19), the value 11.6 is the average, and not the minimum, content of H_2S.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

LABORATORY INVESTIGATION OF FLUME TRACTION AND TRANSPORTATION

Discussion

BY HANS KRAMER, M. AM. SOC. C. E.

HANS KRAMER,[36] M. AM. SOC. C. E. (by letter).[36a]—This paper, based on flume experimentation and correlation of data from the observations of previous investigators, adds another valuable chapter to the field of knowledge regarding the complex phenomena of sand movement. Because of its intimate relation with, and frequent reference to, the writer's previous work[37] in the same field, this discussion will be confined to Part I of Mr. Chang's paper, which treats the subject of "Flume Traction."

The author made use of a horizontal flume bed instead of one inclined to the same slope as the water surface because the necessity for the latter arrangement was considered doubtful. However, from a mathematical analysis of his observed data, Mr. Chang concludes "that the tractive force needed to move débris on a sloping bed parallel to the water surface is only one-half that needed on a horizontal bed." The hypothesis and conclusion appear irreconcilable. Professor Hubert Engels, the pioneer and dean among investigators in this special field, emphasized[38] the importance of uniform flow and the interrelation between bed slope and water-surface slope in experimentation of this nature. It is unfortunate, therefore, that Mr. Chang's observations have been beclouded unnecessarily by the introduction—or lack of elimination—of this important variable.

A similar comment is applicable to the uniformity modulus, *M*. The author states that the uniformity modulus of samples used in his experiments was not determined but, nevertheless, concludes—again by mathematical manipulation—"that the uniformity modulus, *M*, has practically nothing to

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NOTE.—The paper by Y. L. Chang, Esq., was published in November, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[36] Capt., Corps of Engr., U. S. Army; Dist. Engr., U. S. Engr. Office, Conchas Dam, N. Mex.

[36a] Received by the Secretary January 31, 1938.

[37] "Sand Mixtures and Sand Movement in Fluvial Models," by Hans Kramer, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 798.

[38] "Versuche über den Reibungswiderstand zwischen strömenden Wasser und Bettsohle," by H. Engels, *Zeitschrift für Bauwesen*, 1912.

do with the critical tractive force, and could be omitted." Inasmuch as Mr. Chang's experimentation purposely utilized only sands of uniform grain size ($M = 1$), whereas natural sand mixtures vary greatly in their gradation—and in M—it would appear that his extrapolated conclusion as to the effect of the uniformity modulus is beyond the limit of his own observations.

The painstaking determination of the mechanical properties of test samples, including microscopic measurements, deserves commendation. It is regrettable, however, that the determination of grain size by the standard method of sieve analysis was not included. It is believed that the grain size of sand used by other investigators, as listed in Column (6), Table 5, is based on the conventional sieve analysis curve. Inasmuch as standard sieving procedure results in larger apparent sizing than absolute physical measurement, Mr. Chang's size determinations are not directly comparable with those tabulated for other investigators.

It is also noted that Mr. Chang has adopted as a definition for "general movement" a set of conditions which are more comparable to the writer's definition for "medium movement" than to his definition of "general movement." A cursory review of the writer's observed data indicates that the magnitude of the critical tractive force based on his "general movement" is approximately 20% greater than that based on "medium movement." This variance in a basic definition, which forms a criterion for comparison of results, would appear to require the application of a corrective factor in the correlation of observations.

Mr. Chang's experimentation and treatise, while constituting another important step in the diagnosis of the complex phenomena of sand movement, may also be regarded as further proof of the difficulty of reconciling and correlating data observed and conclusions derived by different experimenters under widely divergent conditions. His attempt to simplify the problem by choosing to experiment with uni-granular sand, thereby eliminating the vexing variable of grain-size distribution, is a step in the right direction. However, it is believed that the ultimate, basic solution to this problem—both mathematical and physical—will not be clearly and convincingly obtained until the scientific viewpoint and procedure of the experimental physicist are adopted. The writer is convinced that further research to determine the independent variables which govern the very complex phenomena of sand movement by traction and the fundamental relation between these variables must seek simplification and idealization of the phenomena. To this end, it is suggested that future experimentation with artificial spherical or spheroidal grains—probably of glass—with colors, markings, etc., for statistical identification may develop the simple fundamentals of flume traction without the complicating influences of accidental and uncontrollable variations inherent in the use of natural sand specimens.

Corrections for *Transactions:* Delete the captions for Figs. 12 and 13; insert caption for Fig. 15: "Distribution of Pumice in Suspension (Energy Gradient, $S = 0.000088$)"; and, insert caption for Fig. 16: "Silt Distribution and Total Suspended Load in Stream with Parabolic Velocity Distribution."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

ECONOMIC PIPE SIZES FOR WATER DISTRIBUTION SYSTEMS

Discussion

BY MESSRS. CLINTON L. BOGERT AND J. M. M. GREIG,
CHARLES M. MOWER, JR., AND A. C. MICHAEL

CLINTON L. BOGERT[11] AND J. M. M. GREIG,[12] MEMBERS, AM. SOC. C. E. (by letter).[12a]—The clever formulas derived in Professor Camp's interesting paper exhibit to the writers the danger of carrying formulary regimentation into the realm of practical engineering. The writers question the economy of such formulas in office use. The formulas give a designer the exact diameter to fit a given set of fixed conditions; but, unfortunately, conditions do not remain fixed and, more important, cast-iron pipes are not made in the fractional diameters called for by the economical analysis, the designer being constrained to accept stock sizes which increase in diameters by relatively large steps.

A typical example, first solved by substituting in the formulas, and again by the usual trial methods with a Williams-Hazen slide-rule, for a compound line of two sizes and carrying diminishing quantities, required 2 hr to derive economical sizes of 23.2 in. and 19.0 in., respectively, and less than 10 min by slide-rule to arrive at stock sizes of 24 in. and 18 in., respectively.

Moreover, several of the "constants" assumed by Professor Camp as non-variables in the differentiation, are by no means constant in the practical sense:

(1) He assumes the price of cast-iron pipe as constant; bids taken by the writers' office in November were at least 30 cts per ft lower than they would have been if taken in August, 1937, due to the drop in cast-iron pipe prices;

(2) He assumes the quantity of water as a definite forecast, whereas adverse zoning restrictions, or the advent of a large industry, may upset the "picture"; and,

(3) He assumes that the Williams-Hazen-constant, *C*, is definitely known for a given time and place, whereas the quality of the water or corrective treat-

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NOTE.—The paper by Thomas R. Camp, M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[11] Cons. Engr. (Sanborn & Bogert), New York, N. Y.

[12] Designer (Sanborn & Bogert), New York, N. Y.

[12a] Received by the Secretary January 3, 1938.

ments of the water initiated at some future date may alter the values of C and the entire economic "picture" framed by the formula. The writers grant that these uncertainties also prevail in slide-rule analysis, but are less important as less time is involved.

They would qualify the assumption (see "Introduction") that the cost of elevated storage will not be influenced by changes in pipe sizes. It would not hold good in all cases. A certain water company has 17 tanks and stand-pipes and 41 wells spread over an area of 35 sq miles; some tanks are near wells and some distant. In this system the cost and allocation of future storage is influenced directly by the size and system of mains between one or more tanks and the centers of supply and the centers of demand, which are, of course, not collocated.

Charles M. Mower, Jr.,[13] M. Am. Soc. C. E. (by letter).[13a]—The often used "economic velocity" is not the correct standard for attaining true economy in distribution system design. The author has shown this fact clearly. The economic diameter of a pipe depends upon both discharge and take-offs from the system and upon unit costs of both pumping and pipes. In a system with a pumped supply, the two sources of power loss are loss of head in the pipes and take-offs at higher pressures than are required; but in a gravity system, the head loss in pipes costs nothing. The only losses which add to the cost of the gravity system are take-offs at pressures higher than those required where the excess is provided by pipes that are larger than necessary.

The author's assumption that pumping cost is directly proportional to head and discharge may be questioned since the capital cost of station and equipment and the operator's wages in a particular system are fixed regardless of considerable fluctuation in head and discharge. Since power is the only part of total pumping cost that bears a direct relationship to head and discharge, it would seem more logical to omit all fixed pumping costs from the equation just as the author omitted unpredictable pipe costs, and the cost of hydrant installations which were not affected by pipe sizes. However, in a practical problem, the result is not greatly changed by the use of power costs instead of total pumping cost, because pumping cost decreases much more rapidly with an increase in pipe diameter than pipe cost increases. Pumping cost varies inversely as $D^{4.87}$, whereas the cost of pipe laying varies directly as D and the cost of pipe materials, as $D^{1.55}$.

For the same reason, the total cost for a pipe one size larger than the most economical size is much less than the total cost for a pipe one size smaller. Thus, when computations indicate an odd pipe size, the next larger standard size should be used.

The methods of determining the pipe sizes for best economy for the different classifications of pumped supplies are just as applicable when a design is made on the basis of field tests of an existing system instead of the method suggested by the author, of dividing the system into districts and estimating domestic and fire drafts. In Case II particularly, there would be an advantage in having field measurements of the variations in flow for determining the value of A in Equation (23).

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[13] Asst. Engr., The Pitometer Co., New York, N. Y.

[13a] Received by the Secretary February 21, 1938.

A. C. Michael,[14] Assoc. M. Am. Soc. C. E. (by letter).[14a]—For his attempt to rationalize the economic solution of complicated pipe-design problems, in which there are such a large number of variables, the author deserves to be commended. The mathematical derivations are very cleverly executed, except that certain assumptions and required adjustments might be questioned. The coefficients of the formulas are rather complicated, and, no doubt, considerable estimation would be required for their determination.

For very small systems, where the character of the water demand is simple, and where the piping arrangement is not complex, the method proposed probably could be used to advantage. The writer feels, however, that mathematical analyses of the problems of a water distribution system have a very limited application, and this is particularly true for a system of any size or complexity.

Of the variables considered in the formulas, many are extremely indeterminate. Costs vary over wide ranges; the cost of labor for laying parallel lines of the same diameter has been known to vary 50%, or more. The cost of pumping varies widely with changes in costs of fuel and labor and many other variables. The cost of excavation is usually an unknown quantity until after the work is completed.

The most difficult problem, which the author does not discuss, is the determination of average and maximum flow in a proposed pipe line. If a line is in a newly developed section of the city it is impossible to predict the ultimate water demand to be supplied. In many cases, a proposed pipe line is designed and ready for construction when it is learned that a new industry, using large quantities of water, is about to locate in the area to be served; it then becomes necessary to re-design the proposed pipe line and possibly parts of the existing system. This condition will continue until city zoning and planning ordinances have "teeth." The recent and rapid increase in air-conditioning installations is an example of how conditions change suddenly to cause existing piping systems to become inadequate. The careful development and study of population growth curves, per capita consumption, etc., may often be of little value after a depression in the business cycle has wrecked the trend of the established curves.

In addition to these factors, the variations in the daily load curve must be considered; the ratio of maximum hourly demand to the average daily demand is quite large, at times attaining a value of 3.0 or more. It is conceivable that a pipe having an economic diameter determined by the method described by the author would not have sufficient capacity to supply the maximum hourly demand.

Seldom is the engineer confronted with the problem of designing a complete water-distribution system. These systems grow gradually by the addition to, or extension of, existing lines. The usual problem is to select pipe sizes that will meet average flow conditions with normal pipe velocities, and maximum flow conditions with a respectable pressure at the consumer's outlets. The only suitable method open to the designer is to collect all the information possible, use all the tools and intuition he has at his command, and estimate the future average and maximum demand on the line or system. A size of line may then be selected such that, at maximum flow conditions, the pressure loss will be suitable and the velocity will not exceed a reasonable value.

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[14] Asst. Mech. Engr., Dept. of Water Supply, City of Detroit, Dearborn, Mich.

[14a] Received by the Secretary February 23, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

EARTHQUAKE STRESSES IN AN ARCH DAM

Discussion

BY MESSRS. A. FLORIS, AND CECIL E. PEARCE

A. FLORIS,[6] ESQ. (by letter).[6a]—Earthquake stresses produced in an arch dam slice, fixed or hinged at the abutments, but free to move laterally, are analyzed in this paper. Only the influence of the inertia forces of the arch is taken into account, those of the water behind the dam and of the canyon walls being omitted. Under these simplifying assumptions the seismic analysis of the structure is complete.

However, the question arises as to whether such an analysis can be considered as criterion of the seismic resistance of an arch dam. It is true that buildings are analyzed for earthquake shocks by considering their inertia forces only. These structures are usually free to move in all directions above the ground, the resistance of the air being negligible. In an arch dam the inertia force of water, in view of the lightness of the structure, is an important item. In buildings, the dead and live loads are the gravity forces that mainly determine the design. The weight, the only gravity force exerted by arch dams, is neglected in the stress analysis as unimportant. The available theories pertaining to the influence of water upon the dam, presuppose a straight structure with a vertical up-stream face, true only in the case of a gravity dam.[7] Moreover, the arch dam is enclosed in the narrow canyon between the mass of rock. What will be the effect of all the surrounding inertia forces upon the dam? The present analysis fails to yield an answer.

Bearing these limitations in mind, the paper may be judged as an interesting contribution to the aseismic design of arch dams.

CECIL E. PEARCE,[8] ASSOC. M. AM. SOC. C. E. (by letter).[8a]—The effects of earth shock upon an arch dam are analyzed in this interesting investigation.

NOTE.—The paper by Ivan M. Nelidov and Harold F. von Bergen, Assoc. Members, Am. Soc. C. E., was published in December, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Dipl.-Ing., Los Angeles, Calif.

[6a] Received by the Secretary January 24, 1938.

[7] "Earthquake-Proof Construction of Masonry Dams," by Nagaho Mononobe, M. Am. Soc. C. E., *Proceedings*, World Eng. Congress, Tokyo, 1929, Vol. IX, Pt. I, p. 275; and "Water Pressure on Dams During Earthquakes," by H. M. Westergaard, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 418.

[8] Principal Hydr. Engr., TVA, Knoxville, Tenn.

[8a] Received by the Secretary February 18, 1938.

It has ordinarily been stated by enthusiasts that an arch dam can take an earthquake overload better than any other type because all that results is a slight increase in compression, with no alarming effects in regard to sliding or overturning, and the claim is probably correct. These statements refer to an earthquake in which the dam is accelerated up stream.

However, when the direction of the acceleration is at right angles to the stream, the results are different. The authors have shown that these effects are more serious than the up-stream acceleration usually investigated. This checks the statement made by Mononobe:[9]

> "The effect of earthquake upon ordinary masonry dams is most pronounced when the transverse seismic wave propagates parallel to the longitudinal axis of the dam. The seismic wave causes a forced displacement around the perimeter of the dam's base. If the body of the dam is of one mass, the different parts of the dam's foundation bed exercise relative displacements. This will cause to develop vertical and horizontal crosswise cracks and leakage due to separation of the base of the dam from the bed rock. To remedy these defects and to prevent excessive stresses, vertical crosswise joints or expansion joints may be employed."

Mononobe further suggested that the contraction joints be spaced far enough apart so that the different blocks of the dam can be made stable against accelerations in the direction parallel to the longitudinal axis. In high dams, however, this procedure would conflict with the spacing requirements of contraction joints dictated by shrinkage coincident with the cooling of the concrete.

It would seem that the worst case would be for neither of the two conditions investigated by the authors but for the condition of reservoir empty and an acceleration of the dam down stream. For flood control reservoirs (which should be empty as much as possible), and other reservoirs which are well drawn down part of the year, this case is about as likely to occur as the "reservoir-full" cases investigated. Without computation, it seems plausible that the relatively thin arch dam section would be far less stable against a down-stream acceleration than the top part of a gravity dam, because in this direction there is no arch action in the dam.

It should be emphasized that the formulas developed bear considerable resemblance to those developed by the late William Cain, M. Am. Soc. C. E.[10] The value 2.88 is recognized as the factor for the inclusion of the elastic work of shear.

The writer has computed and checked numerical values for such expressions as: $\phi_1\left(\phi_1 + \frac{\sin 2\,\phi_1}{2}\right)$; $2\sin^2\phi_1$; $2.88\,\phi_1\left(\phi_1 - \frac{\sin 2\,\phi_1}{2}\right)$; $2\sin\phi_1$; $\left(1 - \frac{\sin\phi_1}{\phi_1}\right)$; and, $\left(\frac{\sin\phi_1}{\phi_1} - \cos\phi_1\right)$, for every value of $2\,\phi_1$ from 1° to 180°, by 1° increments which appear in published form.[11] These values are listed in from five to seven decimal places, and are sufficiently accurate for machine computation. Their use shortens the work considerably.

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[9] "Earthquake-Proof Construction of Masonry Dams," by Nagaho Mononobe, M. Am. Soc. C. E., World Eng. Congress, Tokyo, Japan, 1929.

[10] "The Circular Arch Under Normal Loads," *Transactions*, Am. Soc. C. E., Vol. 85 (1922), p. 233.

[11] "The Analysis of Circular Arch Dams by Cain's Method, Including Shear," *Western Construction News*, September 10, 1927.

It should always be borne in mind that although formulas such as those developed by the authors seem somewhat complicated at first glance, essentially, they are simply formulas for finding the bending moment, M (in a horizontal plane), and the horizontal thrust, P, at any point in the arch. With these values once obtained, the stresses may be calculated by the simple beam formula, $S = \frac{P}{A} \pm \frac{M\,c}{I}$, the notation for which needs no explanation.

Corrections for *Transactions:* In Equation (2), delete a in the denominator and the subscript, q, in the numerator; in the second line of Equation (5) insert "+" after the quantity, "$- \phi_1 \frac{r'}{r}$"; change Equation (9) to read "Z" $= \left\{ \frac{3}{4} \sin 2\,\phi_1 + \frac{3\,r' \sin 2\,\phi_1}{2\,r} - \phi_1 \left[\frac{3}{2} \cos 2\,\phi_1 + \phi_1 \sin 2\,\phi_1 + \frac{r'}{r} (2 + \cos 2\,\phi_1) \right] + \left(\frac{2.88\,r'}{r} - 1 \right) \frac{k^2}{r^2} \left[\left(\frac{\sin 2\,\phi_1}{4} - \phi_1 \frac{\cos 2\,\phi_1}{2} \right) \right] \right\}$" $\div$, etc.; Equation (13a) should read "$M = M' + X\,y + Y\,x + M_e$"; Equations (20) and (21) should be renumbered: (20), (21a), (21b), and (21c), respectively; change the parenthesis in Equation (20a) (corrected to Equation (20)) to read: "$\left(\cos \phi - \frac{r'}{r} \cos \theta \right)$"; in Equation (20$b$) (corrected to Equation (21a)) rewrite as follows: "$* * * - \frac{r'}{r} (\sin \phi_1 - \sin \phi)$"; in Equation (26$d$), multiply L by 2; in Equation (28c) change "cos" to "cot"; and, in Line 3 following Equation (30c), change "$\psi = 0$" to "$\psi = 90°$."

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

GRAPHICAL REPRESENTATION OF THE MECHANICAL ANALYSES OF SOILS

Discussion

BY MESSRS. CARL H. KADIE, JR., CARLTON S. PROCTOR, T. T. KNAPPEN, JACOB FELD, AND HOWARD F. PECKWORTH

CARL H. KADIE, JR.,[13] JUN. AM. SOC. C. E. (by letter).[13a]—The author has developed a method of tabulating, in a simple form, a mass of information regarding the nature of the particle-size distribution of a soil sample. With the increasing use of soil classification and soil tests in the investigations of building foundations, earth dams, highway sub-grades, and erosion control, a need has been felt for a simple method of recording pertinent data on soil profiles and maps. Mr. Campbell's grade line, with the deviation therefrom, and the use of the mean diameter, provides a means of supplying all the necessary information regarding the particle-size distribution and eliminates the ambiguous and difficult-to-read symbols generally used in such soil maps and profiles. The symbolic representation of soil classifications, when appearing on drawings which have been reduced in scale, is generally unintelligible. The writer believes that the method of grade lines is a remedy for the aforementioned difficulty, but to be useful it must be standardized. It would seem of little importance which sizes are chosen to correspond to the numbers, 1 through 5, but it is important that all data recorded by this method use the same system.

The need for standardization has been felt by many who have had occasion to read the reports from various engineering offices and Government bureaus. For example, it is of no importance whether the smallest particle size is found to the right or to the left of the mechanical analysis curve. Both conventions will be found in regular use. Probably more curves showing mechanical analyses of soils have been plotted with the small sizes at the left of the sheet, whereas concrete sands and aggregate are generally reversed. There are a great many exceptions to this, however. Although it is a small point, the standardization

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NOTE.—The paper by Frank B. Campbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1938, by Messrs. Donald M. Burmister and A. J. Weinig, Jr.

[13] Asst. Engr., U. S. Bureau of Reclamation, Yuma, Ariz.

[13a] Received by the Secretary February 1, 1938.

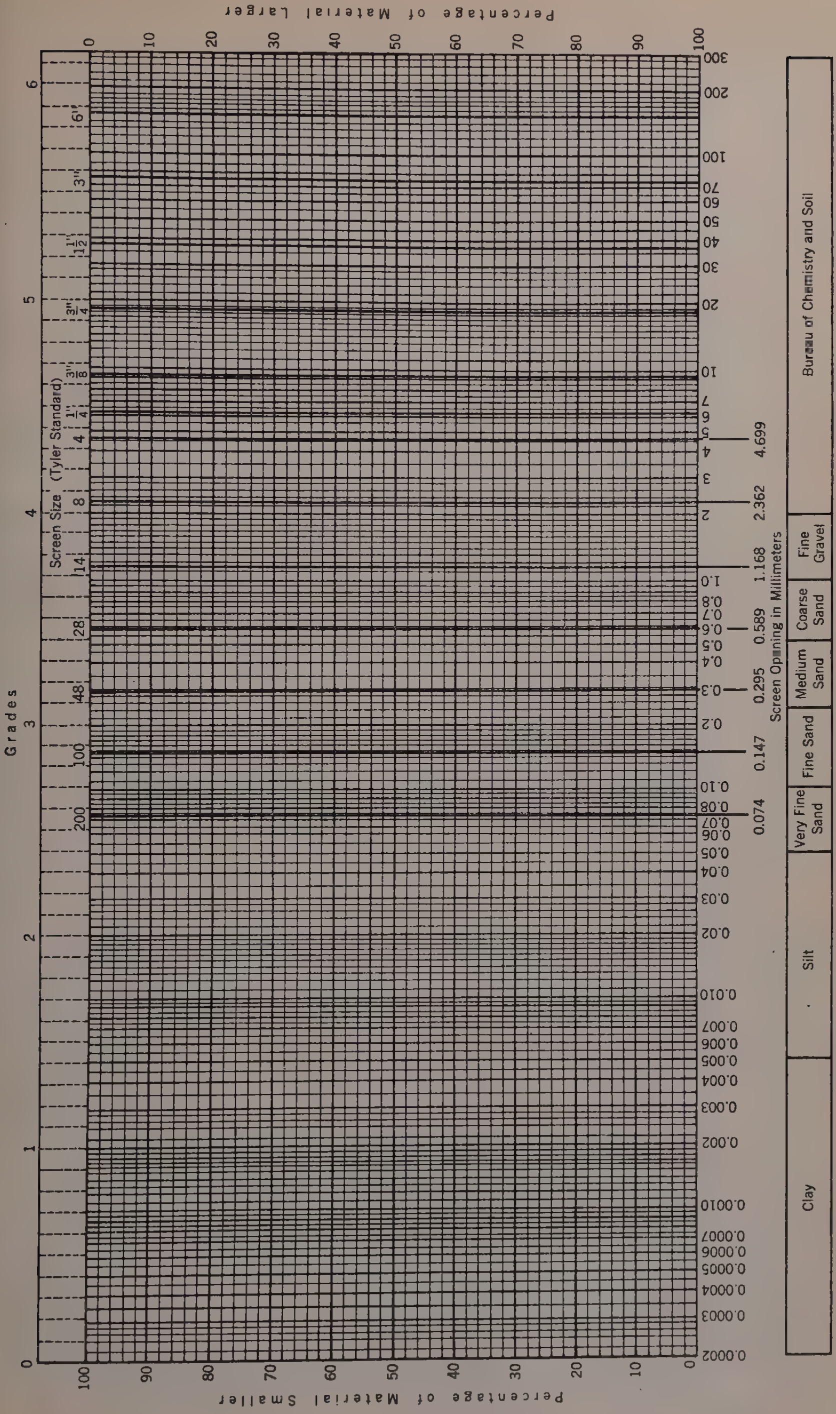
Percentage of Material Larger
0
10
20
30
40
50
60
70
80
90
100
Percentage of Material Smaller
100
90
80
70
60
50
40
30
20
10
0
Grades
0
1
2
3
4
5
6
Screen Size (Tyler Standard)
6"
3"
1½"
¾"
⅜"
¼"
4
8
14
28
48
100
200
300
200
100
70
60
50
40
30
20
10
7
6
5
4
3
2
1.0
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.10
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.010
0.007
0.006
0.005
0.004
0.003
0.002
0.0010
0.0007
0.0006
0.0005
0.0004
0.0003
0.0002
Screen Opening in Millimeters
4.699
2.362
1.168
0.589
0.295
0.147
0.074
Bureau of Chemistry and Soil
Fine Gravel
Coarse Sand
Medium Sand
Fine Sand
Very Fine Sand
Silt
Clay

FIG. 3

of this convention, regardless of the use made of the curve or of the material, seems highly desirable.

The writer does not agree that any material benefit may be derived from the proposed change in the limits of sizes in the various soil classifications. Since clay cannot be defined by size alone, the division between clay and silt must be of an arbitrary nature. Division by assigning a larger size to the upper limit of clay would save time in sedimentation analyses in reaching this point. The time, however, required to reach a particular size decreases with an increase in temperature or specific gravity. Each investigator can readily determine how much time he is justified in spending on the sedimentation analyses.

The United States Bureau of Chemistry and Soils has adopted size limits for each classification of soil. These limits have come into wide use, and in view of the fact that no soil class can be entirely identified by size alone, it seems desirable to adopt these limits as a standard.

The transfer of tabular data on soil separates to a mechanical analysis graph sheet for reproducing its curve, can be facilitated by the simple method shown in Fig. 3. The screens used to accomplish this separation are designated by number and a line is drawn on the graph sheet at the position of the size of their openings. The intermediate lines facilitate the determination of mean size and "grade line deviation." The "grade numbers" are those suggested by Mr. Campbell, and the soil classifications are those of the Bureau of Chemistry and Soils.

CARLTON S. PROCTOR,[14] M. AM. SOC. C. E. (by letter).[14a]—A timely subject is treated in a refreshingly lucid style in this paper. Mr. Campbell is to be complimented for an excellent contribution to a subject on which there is general engineering agreement as to its immediate importance; because, certainly, an engineering reference to a particular type of soil should be subjected to a uniform general interpretation and such is not now the case.

The existing confusion in soil classification standards is largely a confusion in terminology, and a standard classification is to be expected only after the establishment of a standard terminology. The first step in this direction should be agreement as to the number of classifications to apply to each soil group.

Mr. Campbell proposes three classes of sand, whereas the U. S. Bureau of Chemistry and Soils includes four, and the classification listed in Fig. 1 as the "Tyler sieve" group gives five sand classes. A Tyler sieve classification was proposed by William P. Kimball, Assoc. M. Am. Soc. C. E., in 1936, based on the result of a practical experiment.[15] Professor Kimball submitted a series of bottled soil samples of Tyler sieve grain sizes to various experienced foundation engineers and found a close agreement in classification, in terms of silts, very fine, fine, medium, coarse, and very coarse sands, etc. Because the five grades of sand were invariably used and because the variations in classification occurred at the margins of silt to very fine sand, and sand to fine gravel, a

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[14] Cons. Engr. (Moran, Proctor & Freeman), New York, N. Y.

[14a] Received by the Secretary February 1, 1938.

[15] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., June, 1936, Vol. 1, pp. 302 to 305, inclusive.

Tyler sieve classification was proposed. A similar sub-division of sands was presented by T. A. Middlebrooks,[16] Assoc. M. Am. Soc. C. E., which covered essentially the same grain size ranges for each of the five sub-divisions. This classification was reported as having been used at the Fort Peck Dam, in Montana. The five-group sand classification confirms a wealth of experience of practicing foundation engineers in that for most reports received from the field, five classifications of sand are used. Therefore, the single criticism that can be made of Mr. Campbell's paper lies in his use of only three sand classifications and in the fact that the abscissa scale of Fig. 1 does not lend itself to four or five sub-divisions.

The soil classification that will be ultimately accepted as standard must be equally practicable of application in both field and laboratory and must follow the terminology familiar to, and accepted by, the contractor who will be guided by the engineer's plans and specifications. The scale proposed meets this condition except for sand, but the fact that Tyler concrete sieves are normally found in the ready equipment of engineers, architects, and contractors should not be overlooked in any consideration of a standard soil classification.

The importance of agreement in terminology of soil classifications lies in the practical consideration that most soil classifications are, and will probably continue to be, made as a result of experienced visual examination only, occasionally correlated by trial mechanical analysis to a preferred classification. Mechanical analysis of all, or nearly all, the samples in a large exploration project is too time-consuming and expensive for many operations.

The paper, as a whole, is highly commendable. The proposed method of recording and tabulating mean diameter, grade line deviation, and grade is ingenious and may well prove of real practical value. This is a logical development from the work of the late Allen Hazen, M. Am. Soc. C. E., in the field of filter sands.

T. T. KNAPPEN,[17] M. AM. SOC. C. E. (by letter).[17a]—Grain-size distribution graphs give the engineer or technician a picture that is useful in suggesting, in a general manner, what the properties of a soil may be. The author has indicated a possible method of classifying grain-size curves for statistical purposes. Presumably, such classification might prove to be useful in coordinating physical properties with grain-size distribution. The author remarks that the grain-size curve is frequently an indication of type—that is, residual, alluvial, or æolian—and that it may be an indication of maturity. Otherwise, he does not suggest much use for his method of representing grain-size curves numerically. Possibly he can make the value of this method of classification more apparent.

In the past the writer has considered various methods, including several similar to that suggested by the author, for statistical comparison of the properties of soils, but abandoned them as useless. The engineer is interested in many properties of soils in their natural and remoulded states. Some of these properties are: Permeability; cohesion; internal friction; consolidation characteristics; structure; porosity, or void ratio; water content; chemical composition; specific gravity; etc. So many factors other than grain-size distribution are vital in predicting the behavior of a soil under certain conditions, that one wonders if any useful purpose can be served by the proposed method of classification. Its value must be less than the grain-size curve itself since it is only an approximate method of describing such a curve. To illustrate, the 50% size gives little if any indication of permeability, and even the 10% and 20% sizes may be of little use for this purpose. Unless there are more values than are apparent, it is the writer's belief that the introduction of such a method of classification would be only one more confusing factor in an already "muddled" field.

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[16] *Proceedings*, International Conference on Soil Mechanics and Foundation Eng., Harvard Univ., Cambridge, Mass., June, 1936, Vol. 1, p. 145.

[17] Associate, Parsons, Klapp, Brinckerhoff & Douglas, New York, N. Y.

[17a] Received by the Secretary February 8, 1938.

With his tabulation of the nomenclature used for various soil separates, the author illustrates the confusion now present in this field. He suggests still another method of classifying the separates. Arthur Casagrande, Assoc. M. Am. Soc. C. E., once suggested to the writer that these various efforts to apply well known soil names to these fractions were not only idle but also harmful. Remembering this remark the writer consulted a dictionary to see what these words meant to the authorities on language. Clay is defined[18] as,

> "A widely distributed earth, plastic and tenacious when moist and hardening when baked, consisting of pure kaolin or, more commonly, of a mixture of kaolin with more or less finely eroded material, chiefly quartz, feldspar, and mica. * * * Dry clay consists essentially of a compound of alumina, silica, and the elements of water. When it is subjected to a baking heat water is formed and passes off as steam and a material quite different from the original clay is left behind. It is very hard, and its powder, mixed with water, is not plastic. * * *"

Silt is defined as: "(*a*) Mud or fine earth suspended in running or standing water; (*b*) a deposit of such mud or fine earth." Sand is "a loose material consisting of small but easily distinguishable grains (most commonly of quartz), resulting from the disintegration of rocks. * * *"

It might be best to forget the various attempts to apply these names to soil fractions and retain them as a common language for the engineer and layman. Are not such terms as "blow sand," "sticky clay," "soft silt," "loess," "gumbo," "coarse sand," "hard-pan," "boulder clay," etc., more descriptive for conveying to the engineer or layman the general properties of a soil than "D 0.022-G 3.9-Grade line deviation + 0.021"?

JACOB FELD,[19] M. AM. SOC. C. E. (by letter).[19a]—Separation of soil particles by means of square mesh screens is the oldest method of analysis for the purpose of classification. The author's proposed standardization of sizes to match grades covering the usual soil fraction names is complete and should be adopted. However, it must be carefully noted that the separation by screens does not distinguish between particles of different shapes. It is perfectly possible to have identical gradation curves, both for sizes and for percentages in each grade, for two very dissimilar soils. The differences in physical action of large grains, above the grade of fine sand, of different shapes can be disregarded. For smaller grains, ratio of surface area to mass becomes an important factor, because of the importance of surface tension and other cohesive forces in small-grained soils.

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[18] Webster's New International Dictionary, G. & C. Merriam Co., 1930.

[19] Cons. Engr., New York, N. Y.

[19a] Received by the Secretary February 10, 1938.

The effect of shape can best be pictured by the following tabulation of grains, all of which will be retained on a square-mesh screen with a width of opening slightly smaller than unity:

(1) A cube with unit side, has a volume of one, surface area of 6, and a ratio of surface to volume of 6.

(2) The cube split into halves by a plane parallel to, and midway between, two opposite faces; each grain has a volume of 0.5, a surface area of 4, and a ratio of surface to volume of 8.

(3) The cube split into halves by a plane, through two opposite edges; each grain has a volume of 0.5, a surface area of 4.41, and a ratio of surface to volume of 8.8.

(4) The cube split into three equal parts by two planes parallel to opposite faces; each grain has a volume of $\frac{1}{3}$, a surface area of 3.33, and a ratio of surface to volume of 10.

(5) The cube split into three equal parts by planes joining one corner with two edges meeting at the opposite corner (forming pyramids with a square base, two normal and two oblique faces); each grain has a volume of $\frac{1}{3}$, a surface area of 4.83, and a ratio of surface to volume of 14.5, almost 2.5 times that of the cube.

The process can be continued indefinitely, but the general picture shows why sand and clay or basically cubical and flat grains act so differently. When particles are in the silt and clay grades, shape is at least equal in importance with size.

HOWARD F. PECKWORTH,[20] M. AM. SOC. C. E. (by letter).[20a]—The concentration of graphical representation of grain-sized distribution into a recognized standard, as presented by Mr. Campbell, is an excellent proposal. If experience shows that his proposed system meets with general approval an effort would be made to secure its adoption as a standard.

As the author proposes in part to fix the definition of "sand," "silt," "clay," and "colloid" in relation to grain size, and as it is necessary to distinguish between the popular usage of these words and their uses in relation to grain size, it is unfortunate that the meanings of the words, "silt" and "clay," are sometimes in such sharp disagreement with their meanings as proposed and as used in the grain-size distribution charts.

An example of this disagreement was demonstrated by a very thin bituminous coal seam running below a borrow-pit which supplied material for a rolled-earth dam. When the steam shovel, digging into the borrow-pit, broke through into this entirely unsuitable coal seam it picked up fine-grained, broken, coal which, when examined with the earth, fell between the grain sizes, 0.0006 and 0.06 mm; and yet this coal could never be called clay or silt by any stretch of the imagination.

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[20] Res. Engr., Birmingham Industrial Water Supply Comm., Birmingham, Ala.

[20a] Received by the Secretary February 17, 1938.

Another example is presented by fines, dust, and waste from a sandstone crushing operation being piled next to a borrow-pit to be used for earth for a rolled-earth dam. When wet it was sometimes difficult to distinguish between the material in the borrow-pit and the dust from the waste pile. Some of the material in the waste pile would fall between the sizes, 0.0006 and 0.06 mm, on the grain-size distribution chart; and yet it was 90% pure silica particles, which again could not be called clay or silt.

If this proposed classification is to be adopted as a standard, the writer would like to suggest that the names of the gradations be changed from "sand," "silt," "clay," and "colloid," to "sand," "fines," "superfines," and "colloid," or some similar wording that would not bring such a sharp disagreement between the popular dictionary usage of the words, "clay" and "silt," with materials actually dealt with in soil mechanics other than clay and silt which nevertheless fall in sizes between 0.0006 and 0.06 mm.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

ECONOMIC ASPECTS OF ENERGY GENERATION A SYMPOSIUM

Discussion

By Messrs. M. M. Samuels, and E. W. Kramer

M. M. Samuels,[40] Esq. (by letter).[40a]—In presenting this discussion the writer assumes that the Symposium was not intended to prove or to disprove any political theories or opinions, but that it was intended as an engineering discussion, whose purpose can never be anything but fact finding.

Attention may well be called first to the surprising diversity of terminology among the authors of the various papers. If every one were to continue using his own favorite terms, engineers would soon be in the position of those who built the tower of Babel. True, many definitions have not yet been established as American Standards Association (ASA) standards; but the proposed ASA standards have been approved by all the respective sub-committees of the various societies and should be used by engineers. Thus, Mr. Orrok speaks of "use factor" and Mr. Justin of "capacity factor," both meaning what the proposed ASA standards term the "plant factor." Mr. Justin has his own definition for "utilization factor," whereas the proposed ASA standards define the utilization factor as the ratio of peak to capacity. Those who have served on standardization committees know that frequently a term which appears at first as the most obvious has to be dropped for one reason or another. Considerable confusion will be avoided if all engineers will speak the same language, even if some of the terms are not to their individual liking.

Considerable confusion in the discussion is brought about by a diversity of appreciation of the significance of the various factors in the cost comparison. Credit is due to Mr. Orrok for dropping the method, previously used by him,

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Note.—The Symposium on Economic Aspects of Energy Generation was presented at the meeting of the Society and at the Joint Meeting of the Power and Engineering-Economics and Finance Divisions, Pittsburgh, Pa., October 14, 1936, and published in the December, 1937, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: February, 1938, by Messrs. W. S. Finlay, Jr., R. M. Riegel, Ralph Bennett, J. M. Mousson, Ben C. Sprague, V. M. Marquis, R. L. Sackett, D. J. McCormack, and I. E. Moultrop.

[40] With Federal Power Comm., Regional Office, New York, N. Y. Any opinions expressed herein are the writer's own and have neither been approved nor disapproved by the Federal Power Commission.

[40a] Received by the Secretary January 22, 1938.

of setting up cost comparisons on a load-factor basis and substituting for it a comparison on a plant-factor basis. Mr. Justin, too, sets up cost comparisons on the plant-factor basis. Mr. Sporn, however, presents his cost data on the load-factor basis. Although other important conditions which would give the data definite meaning are no doubt known to him, to the reader such data are meaningless. The load factor is the ratio of the average load to the peak; it is not even necessary to know the scale of the load curve to establish it. A plant of unknown size may have a peak of 90 000 kw and an average load of 45 000 kw, resulting in a load factor of 50 per cent. The same plant may have a peak of 9 000 kw and an average load of 4 500 kw, resulting also in a load factor of 50%; but the fixed charges per kilowatt-hour in the second case will be ten times as high as those in the first case. Thus, for the same power-house, and the same load factor, the fixed charges per kilowatt-hour can be made to be almost anything any one desires. Why not follow the example of Mr. Orrok and Mr. Justin and set up the comparisons on a plant-factor basis? but if load factor it must be, the data presented by Mr. Sporn would become of great value if he were to state the capacities and peaks to which the various load factors relate.

There are many other uncertainties in cost set-ups on the basis of load factor. For instance, the difference between a load factor computed from the instantaneous peak and one computed from the 60-min peak may be as much as 20 per cent. Possibly, Mr. Sporn's curves and figures are based on the assumption that peak is equivalent to capacity, in which case the load factor would be equivalent to the plant factor, and the curves and figures would assume a definite meaning, even if in reality there is scarcely a plant which, in any one year, to say nothing of the average over the life of the plant, will have a peak which is exactly equal to the capacity.

Even when comparisons are related to plant factor great caution is called for. Only in the case of an isolated plant can a comparison be made between hydro-electric and steam on the basis of the same plant factor for both. With power systems as they exist to-day, such an isolated plant situation will scarcely ever present itself. Generally, there will be a system which can absorb all the energy that a hydro-electric plant can generate, as indicated by Mr. Justin. For such cases higher plant factors must be assumed for hydro-electric than for steam. Table 6 shows that on the whole, in the United States, the plant factors of hydro-electric plants have been considerably higher than those of steam plants, which means that on the average considerably more kilowatt-hours per annum per kilowatt of installed capacity are generated by hydro-electric plants than by steam plants. This definitely

TABLE 6.—APPROXIMATE OVER-ALL PLANT FACTORS* OF THE ELECTRIC LIGHT AND POWER INDUSTRY IN THE UNITED STATES

Description	1926	1927	1928	1929	1930	1931	1932	1933	1934	1935	1936	1937
All hydro plants	45	46	49	48	41	35	39	40	39	45	45	48
All steam plants	30	29	29	31	29	28	21	22	25	26	32	34

* Percentages computed from basic data in *Electrical World*, January 15, 1938.

indicates how little value there is in discussing such a subject in a general way. Rational plant-factor assumptions can only be made in given cases, and even then it is of great importance not to overlook the fact that the plant factor on which any cost comparison is made must be an average throughout the entire life of the plant.

It is to be noted that the various authors in this Symposium use different percentages for fixed charges, making it difficult to compare their resulting costs. In reports published elsewhere the differences are even greater than they are in this Symposium. In many cases these variations are brought about by the fact that the several authors include different items under the term, without stating definitely what these items are. To the banker, the term "fixed charges" generally means only interest and amortization of long-term debt discount and expense. For railroads, the Interstate Commerce Commission includes also rentals. In modern utility accounting, the term is not used at all. As related to electric utilities, it is only used as a convenience in estimates, and in this case the diversity is considerable. Some include even income tax and others exclude even depreciation. Therefore, each author, when using the term, should state which items he includes, as does Mr. Orrok in his paper. The cost of money may vary considerably. The depreciation depends on the assumed life of the plant, which, in the case of new steam plants to be built, is to a great extent a matter of design policy. The assumptions vary from 15 yr to 30 yr. On the basis of straight-line depreciation this would represent a variation of one of the items in the fixed charges from $6\frac{2}{3}\%$ to $3\frac{1}{3}\%$ per annum. In 1920, the U. S. Geological Survey reports a total of 2 598 steam plants in the United States, and in 1935, only 1 491. Since the 1935 figure includes a considerable number of new plants, and since units have been removed from the remaining plants, it is evident that the great mortality of steam plants would possibly make a design policy of 15-yr life for steam plants rational. On the other hand, in 1920, the U. S. Geological Survey reports a total of 1 172 hydro-electric plants in the United States, and in 1935 as many as 1 523, an indication of a very low mortality. Thus, it is evident that, unless a definite case is at hand and all the details are presented so that they can be analyzed and criticized, the discussion will not mean a great deal.

Another feature which is scarcely ever mentioned is the comparative reliability of service. It costs more to render good service than it does to render poor service, and the cost per kilowatt-hour on a system having two outages per year cannot be fairly compared with that of one having twenty outages per year. Is a steam plant at load center more dependable than a remote water-power station with a transmission line; and what kind of a transmission line—single circuit, double circuit on the same structures, double circuit on separate structures, etc? All these features enter into cost, and unless they are known and included in the cost comparison, such comparison means very little.

As to the individual papers: Mr. Orrok's fine presentation of the progress made in fuel economy would have been much more nearly complete if figures had been presented to show the additional fixed charges involved in the higher fuel efficiency. Mr. A. E. Knowlton recently presented a tabulation[41] of large plants from which it would appear that, above a certain point, the saving in fuel brought about by higher steam pressure is offset by higher fixed charges. At the present stage of development such a point should not be overlooked.

Mr. Rogers, in his discussion of the frequency problem, fails to mention that according to Dr. W. M. White[42] this difficulty may be solved in the future by direct-current transmission.

Even if the Symposium is entitled "Economic Aspects," it would seem necessary at this time to sound a warning that, in the search for economy, reliability of service should not be overlooked. Recent widespread outages throughout the country seem to indicate that "Btu-chasing" has reduced the reliability of service of many an important inter-connected system to a level not much higher than that of an isolated little Diesel plant. Mr. Justin's paper seems to reflect the philosophy of "Btu-chasing" regardless of reliability of service.

The first sentence in Mr. Sporn's paper, calling attention to the fact that the cost of generation is only a small part of the cost of a kilowatt-hour delivered to a customer, is to the writer's mind the most important statement of his paper and possibly of the entire Symposium. Actually, the problem of deciding in an individual case whether a steam plant or a hydro-electric plant should be built, has now become a comparatively simple (although at times lengthy) problem of power engineering. It is indeed surprising that there is still so much general discussion on the subject. It is no secret that among engineers a general discussion of the question of "steam *versus* hydro," when no definite case is at hand, is considered a waste of time. This is not so with distribution. Only a few years ago engineers frowned upon distribution as being "low-brow" engineering, or no engineering at all; it was considered a job for the line foreman and storekeeper. More recently, however, engineers have been devoting considerable time to the study of distribution problems, and distribution has become respectable, even "high-brow," engineering. The results are astonishing. Definite problems have been crystallized and partly solved. Mathematical analysis has been applied to distribution studies, and the history of electrical engineering shows that when mathematical analysis is superimposed on operating experience, fine results are inevitable. It may be safely predicted that before long distribution engineering will be the most important part of electrical engineering. Professors may even consider the advisability of devoting a little less attention to machine design and more to the physical and mathematical fundamentals of distribution. The previously mentioned fault in Mr. Sporn's paper, of basing cost on load factor, is particularly evident in Figs. 33 and 34, which present a comparison of cost between shipping energy in the form of coal on the one hand and electricity on the other. It is unfortunate that these important comparisons are meaningless. No doubt, Mr. Sporn will be able to supplement curves on a plant-factor basis and thus give this important set of data a definite meaning.

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[41] *Electrical World,* October 9, 1937.

[42] "Technological Trends and National Policy," National Resources Committee, 1937, p. 281.

Mr. Sporn's expression of doubt about the economy of transmission over very long distances deserves serious attention, particularly because he is no doubt one of the outstanding and most outspoken authorities on transmission problems. Here, too, it would have been helpful if comments had been included about comparative reliability of service.

It would be interesting to know what method of weighting was applied in establishing the average transmission distance on the specific system as 60 miles, and also what the longest distance is, and what, if any, sad experiences were had with this longest distance?

Mr. Freeman's paper presents definite statements and questions without presenting a single figure to substantiate the statements or the reasons for the questions. It is impossible, therefore, to discuss this paper. One of his statements, however, is of such great importance that it deserves further elaboration: "The old economic problem was how to produce enough; the new problem is how to dispose of all that can be produced." As related to electric energy, the problem is how to fill out the load curve and thus reduce fixed charges per kilowatt-hour—in other words, how to increase the load factor for a given capacity, which, again, is another way of asking how to increase the plant factor. This problem has recently become of even greater importance A few years ago it was generally hoped that the valleys in the load curves would soon fill out and that the difference between peak and average would become less drastic. Recent indications are that on many systems the opposite has happened—the percentage increase in peak has been greater than the percentage increase in consumption. Possibly this is due to shorter labor hours in industry and the speeding up of industrial production during these short hours. Even air-conditioning, which was expected to contribute considerably to load-curve improvement, is in many cases peak load. Air-conditioning has been used thus far mostly in industrial and large commercial establishments.

This problem cannot be solved by engineering alone; it must be solved by more modern business methods and more competent salesmanship. Promotional rates at incremental costs for off-peak loads have shown good results wherever they have been tried out, and where modern sales methods were used. However, the "energy salesman" has not yet entered the home. It is a safe guess that none of the readers of this discussion was ever approached by a salesman who tried to sell him the idea of using a few more kilowatt-hours per month. It is also a safe guess that a good salesman could have sold him those few additional kilowatt-hours per month. Of course, in order to sell, a salesman must at least know his own sales prices. With the present complex rate schedules this is difficult. What is needed, therefore, is, first, a simplification of rate schedules so that the customer can check his bill; and second, a drive by experienced salesmen (possibly men who have had experience in the selling of commodities against competition) to encourage the use of off-peak energy. This, no doubt, is by far the most important social aspect of power at the present moment, and, at the same time, the most important economic aspect.

E. W. Kramer,[43] M. Am. Soc. C. E. (by letter).[43a]—There seems to be a rather common tendency among engineers in discussing the comparative merits of hydro-electric and steam power plants, to make statements that are entirely too general—even when it is assumed, as in the paper by Mr. Sporn, that the remarks apply only to the United States. The following statements are quoted from Mr. Sporn's paper:

> "It covers the economics of steam power *versus* water power and shows that, in most cases, present-day steam power can be produced more cheaply than present-day water power. * * *"

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> "It must be borne in mind that the costs are presented from a general viewpoint. * * *"

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> "* * * that hydro-electric plants, in general, as a source of primary power, are out of the economic range. The true economic function of hydro-electric power is, without doubt, to supplement steam power. * * *"

* * * *

> "It appears definite that future power requirements will be met most economically by steam plants, located at a reasonable distance from load centers. * * *"

It is believed that these quotations, although taken from their setting, represent in part what Mr. Sporn purports to prove in regard to the future of hydro-electric construction. It is the opinion of the writer that these statements are entirely too broad. They would be almost 100% wrong for the Northwestern States, such as Washington, Oregon, and Idaho, where the steam power generated is only about 25% of the total and is economical only for the reason that it makes use of the waste products from saw-mills or has value as stand-by service. The statements would be very questionable when applied to California, for the reason that there are many undeveloped water-power sites of large capacity in California that could compete successfully with steam-electric plants using natural gas or a gas-oil combination as fuel. Finally, the writer would question the general applicability of the statements in the Rocky Mountain States, in the Northeastern States, or in the Southeastern States.

In Table 3, Mr. Sporn shows very high kilowatt-hour costs for a number of hydro-electric projects under Federal Power Commission licenses. Examination indicates that he may have been led to his conclusions by applying an extremely high annual rate to the claimed costs given in order to determine the annual fixed charges. The rate used by Mr. Sporn is 10.75%, but nothing is given in the paper to show how this rate was established. Because the major part of the annual cost of producing hydro-electric power is the fixed charge, the selection of a proper rate is extremely important. If the rate selected is too high, it will, of course, tend very strongly to lead to the conclusion that hydro-electric power is more costly than steam, since the investment in steam is relatively less and the operating cost more. It is the writer's opinion that a well-established, conservatively financed and managed utility with a large and growing market should be able to hold its fixed charges on hydro-electric projects (not including fixed charges on transmission and distribution) to 8% or less. The following build-up of an annual rate, which the writer believes would be applicable to California, is presented for a hydro-electric project with an installation of 50 000 kw, or more, and for a head exceeding 300 ft, where a good part of the investment was in dams, tunnels, and penstocks:

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[43] Regional Director, Federal Power Comm., Region 5, San Francisco, Calif. Any opinions expressed are the writer's own and are not to be taken as official expressions of the Federal Power Commission.

[43a] Received by the Secretary January 31, 1938.

	Percentage
Cost of money	5.5
Depreciation	0.55
Insurance and casualty reserves	0.12
Local taxes	0.7
Total	6.87

It is assumed in this case that no transmission is included, since it is understood that Mr. Sporn does not include transmission in his costs in Table 3. He deals with transmission costs later.

The depreciation figure is determined on a sinking-fund basis, interest at 5.5%, and 45-yr life. The life of the tunnels and dams, of course, will greatly exceed this, although it is probable that the life of the machinery may be less.

Justine and Mervine[44] give 6.7% and 11.5% as the usual minimum and maximum total annual rates for fixed charges, and their "illustrative cases" are based on a "typical" rate of 9 per cent. It will be noted that the writer's estimate is slightly in excess of the minimum rate given, somewhat less than the typical rate, and considerably less than the maximum. The rate used by Mr. Sporn is almost the maximum rate, and it certainly is not generally applicable to the plants given in Table 3.

The principal item, the interest rate, should not vary greatly throughout the United States for a large utility which has been conservatively financed and managed. Possibly 6% might be used. A slightly higher rate for depreciation than the 0.55% used by the writer could be used for low-head plants, where the cost of machinery and equipment represented a larger fraction of the total cost. However, assuming a much shorter life and taking obsolescence into consideration also, this item should not exceed 1% on a sinking-fund basis. This would mean an average life of slightly more than 30 yr with money at 6 per cent.

The operating and maintenance charge should not exceed $0.80 per kw-yr. This is equivalent to the rate of 0.4% on the investment for a plant costing $200 per installed kilowatt, and would bring the total annual rate (fixed charges plus operating and maintenance cost, but not including general and administrative expenses) to 7.27 per cent. The writer is somewhat familiar with the Western plants listed in Table 3 and can see no reason why a much higher rate should be applied to the capital cost of these plants if conservatively financed. It is evident that the application of this annual rate to the other hydro-electric projects shown in Table 3 will lead to very different conclusions from those of Mr. Sporn.

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[44] "Power Supply Economics," p. 153.

The foregoing discussion indicates strongly the importance of financing hydro-electric projects at the lowest possible rate, since a difference of only 1% in the interest rate will represent a difference of approximately 12% in the total annual cost and might easily be the deciding factor in determining the economic feasibility of the project.

Under the heading, "Trends in Hydro-Electric Installations," Mr. Sporn comments that the most economical sites were developed long ago, and that those which have been exploited recently have, in general, been less attractive. The inference from this statement, namely, that the sites still to be developed are even less desirable, is possibly applicable in many localities to projects requiring a medium amount of capital investment. The statement, while possibly still true to some extent, is misleading if applied too generally or to large projects (requiring investments of, say, $40 000 000, or more, to use a rough figure) such as the Boulder, Bonneville, or Grand Coulee projects in the West, or the undeveloped projects on the St. Lawrence River in the East, and on some of the larger streams in other parts of the United States. In many cases, such projects have not heretofore been undertaken on account of their size.

The writer does not wish to bring into this discussion the advantages or disadvantages of the construction of large hydro-electric projects by the Federal Government. However, the fact that the Government used a 4% interest rate in determining the charge for power from the Boulder project, and can undoubtedly borrow money at a lesser rate, should not be overlooked in considering the feasibility and probability of the construction of extremely large projects requiring an investment that, because of its magnitude and because of the many uncertainties involved, such as the growth of load to absorb the large output, could not be financed by a utility—or for that matter by a municipality, or even a State. Sound Federal financing can spread the necessary returns to recover the investment, including interest, over a long period to compensate for small initial returns. Using 4% as the cost of money, and adjusting the interest and depreciation allowances accordingly, the annual rate for fixed charges given earlier in this discussion reduces to 5.65%, which is only little more than one-half the rate used by Mr. Sporn.

It is the writer's opinion that each hydro-electric and each steam-electric project necessarily has to be considered carefully on its merits—that no general assumptions can be made as to their relative advantages or disadvantages. As a general rule, they are at their best when inter-connected. The question as to which one will supplement the other will depend on the relative magnitudes of each type connected to the system, and this relation, in turn, will vary according to the locality.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RELATIVE FLEXURE FACTORS FOR ANALYZING CONTINUOUS STRUCTURES

Discussion

BY MESSRS. FREDERICK SHAPIRO, AND DEAN F. PETERSON, JR.

FREDERICK SHAPIRO,[7] JUN. AM. SOC. C. E. (by letter).[7a]—In the conclusion to his excellent paper, Mr. Stewart seems too enthusiastic over the advantages of his method over that of end-moment distribution.

The writer, who has made an extensive study of the various methods of rigid-frame analysis, feels that for every problem there is at least one system of solution which is superior to others for speed in computation, or for accuracy, or for both. For example, in problems in which an approximate answer is desired quickly for a given loading, the Hardy Cross method[4] is excellent; for a short series of continuous beams, the theorem of three moments is superb, especially when modified by certain short-cuts; and for a rigid-frame bridge with curved beam, the method[8] used by A. G. Hayden, M. Am. Soc. C. E., is very convenient. For the problem of the multi-span rigid-frame bridge with variable moment of inertia which is solved in the paper, the method of transmission coefficients introduced by R. G. Brumfield,[9] M. Am. Soc. C. E., is probably a better one. For those who prefer graphical methods, conjugate points[10] offer an excellent solution to the continuous beam problems. The Beggs' deformeter[11] is ideal for a mechanical solution.

An objection might be raised to some of these methods because they involve the use of formulas. Formulas are intended as short-cuts; they express in one brief mathematical sentence what might occupy several paragraphs of written thought. The devices generally used to avoid them may be likened to going

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NOTE.—The paper by Ralph W. Stewart, M. Am. Soc. C. E., was published in January, 1938, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[7] Draftsman, Gibbs & Hill, Inc., Cons. Engrs., New York, N. Y.

[7a] Received by the Secretary January 19, 1938.

[4] "Analysis of Continuous Frames for Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[8] "The Rigid Frame Bridge," by Arthur G. Hayden, John Wiley & Sons, 1931.

[9] "Moving Loads on Beams with Restrained Ends," by R. G. Brumfield. (Complete manuscript filed for reference in Engineering Societies Library, 33 West 39th Street, New York, N. Y.)

[10] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 1.

[11] *Loc. cit.*, Vol. 88 (1925), p. 1208.

around the block to reach next door. It might be claimed that some of these methods do not give a picture of what is happening to the structure as computation progresses. Is it not easy and simple to put a coin in a slot-machine, pull a lever, and take what is desired from the proper compartment? There is a great advantage in applying this slot-machine principle to the solution of rigid frames in that mistakes are limited to arithmetical ones. The method of transmission coefficients,[12] in the writer's opinion, is one of the best for general application; not only does it give a good picture of the structure at all times, if the computer will pause to look, but it also "grinds" out the solution, if the computer cannot take time to stop.

If only one system of analysis is to be used, it is a matter of personal opinion what method should be selected; but actually different methods handle certain problems more advantageously than others, depending on the purpose in view.

DEAN F. PETERSON, JR.,[13] JUN. AM. SOC. C. E. (by letter).[13a]—The extension of the moment area principle, by the use of flexure factors, seems to the writer to have widespread application to continuous structures. The criterion for its application, as with the other so-called "special methods," is: (1) That the members have essentially straight center axes; and (2) that the deformations are essentially flexural.

Although actual influence lines for moment or shear at any section are not generally necessary to analysis, when cases occur in which they are desirable, they may be constructed directly from the method of flexure factors and the use of the Müller-Breslau relation between influence lines and deflection curves.[14] For a moment at Point *B* (see Fig. 8(*b*)), a unit rotation of the axes is inserted and the flexure factors are written. Span *A B* is solved since $M_{AB} = M_{BC}$. The unit rotation is represented by the number, 31 + 68 = 99. The actual end rotations of the various members may be found by dividing through by 99. The ordinate of the influence line may be found graphically, with the aid of the funicular polygon, or analytically, by a convenient, self-checking table. It is only necessary to construct the deflection curve of each member separately. To do this the length of the member may be divided into a finite number of elementary lengths, Δl. Then, the rotation in the length, Δl, is $\frac{M\,\Delta l}{E\,I}$. This rotation, with sufficient accuracy, may be applied at the center of the length, Δl. Then the designer, knowing the end slopes of each member, traverses these rotations to form the deflection curve. Any accuracy desired may be attained by choosing a sufficiently small value of Δl. If one radian is taken as the initial rotation, the scale of the influence ordinate, in foot-pounds, will be the same as the scale of the abscissa, in feet. If, to begin with, some other rotation is used at the section, the scale of the ordinate will be equal to the initial rotation, in radians. In the graphical method the rotations, $\frac{M\,\Delta l}{E\,I}$, are plotted as vertical forces, *A B*, *B C*, etc., on the equivalent

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[12] *Proceedings*, Am. Soc. C. E., March, 1937, p. 638.

[13] Troy, New York.

[13a] Received by the Secretary February 3, 1938.

[14] For discussion on Müller-Breslau relation see "Structural Theory," by Hale Sutherland, and Harry Lake Bowman, Members, Am. Soc. C. E., Article 10–4.

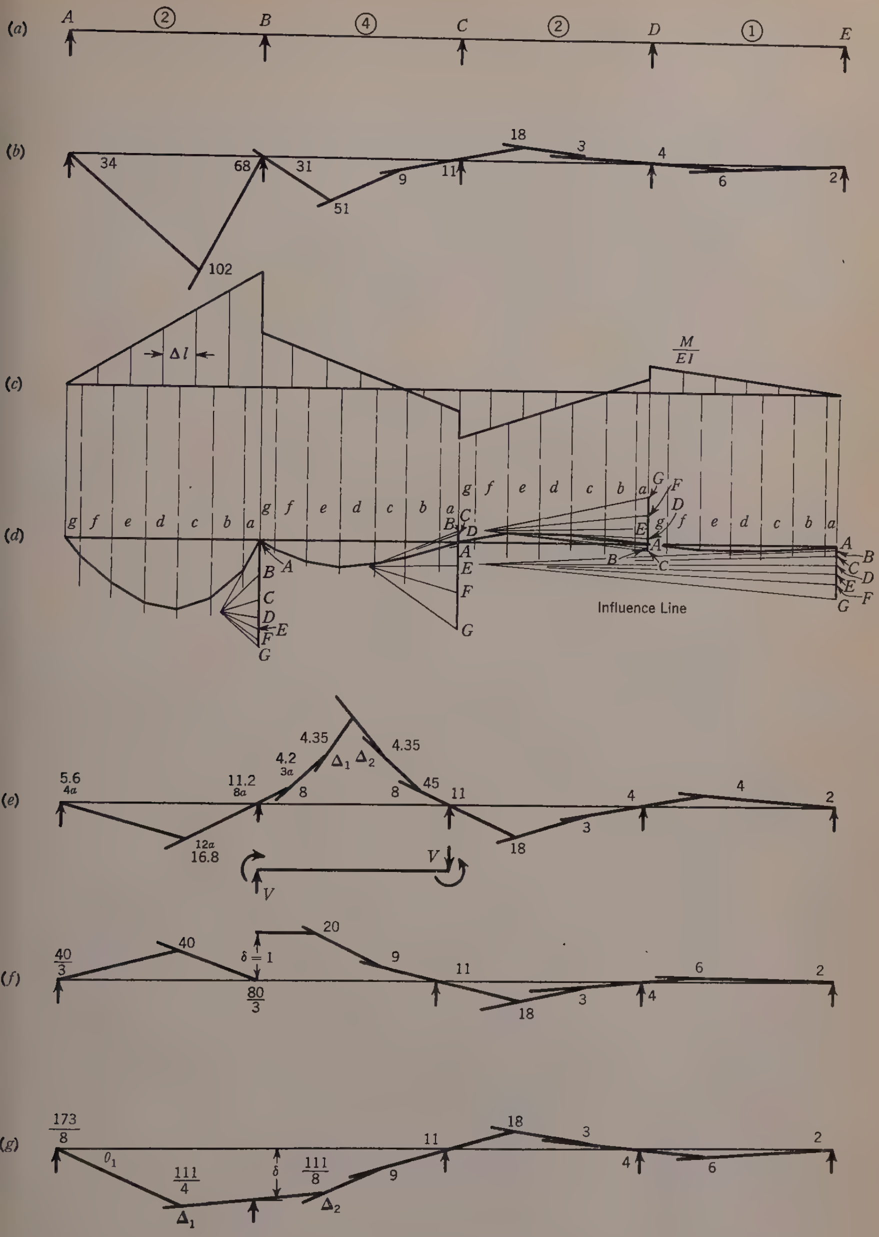
(a)
A
②
B
④
C
②
D
①
E
(b)
34
68
31
51
102
9
11
18
3
4
6
2
(c)
Δl
M/EI
(d)
g f e d c b a
A B C D E F G
Influence Line
(e)
5.6
4a
11.2
8a
12a
16.8
4.2
3a
8
4.35
Δ₁
Δ₂
4.35
45
8
11
18
3
4
4
2
V
V
(f)
40/3
40
δ = 1
80/3
20
9
11
18
3
4
6
2
(g)
173/8
θ₁
111/4
Δ₁
δ
111/8
Δ₂
9
11
18
3
4
6
2

FIG. 8

force diagram. It is not necessary that the true value of $\frac{M\,\Delta l}{E\,I}$, or that the actual moments for the known end slopes, be computed, because the rotations, *A B*, *B C*, etc., may be plotted on the equivalent force diagram to any scale desired in any one span and, therefore, need only be relative. All that is required is that the rays on the equivalent force polygon, through the points, *A* and *G*, be parallel to the end tangents of the beam (see Fig. 8(*d*)).

The influence line for moment at the center of Span *B C* may be drawn (see Fig. 8(*e*)) almost as readily. The flexure factors are written from each direction up to 4.5 on the right and $3a$ on the left, in which a is a proportionality factor to balance the rotations from each direction. This factor may be evaluated since $\Delta_1 = \Delta_2$ (because the moments producing them are equal) and $3 \times 8a + 2 \times 3a + \Delta_1 = 3 \times 11 + 4.5 \times 2 + \Delta_2$ (since the deflection is the same, approaching from the right or from the left). Thus a is found to be $\frac{7}{5}$ and it is possible to express the rotations on each side as comparative units; Δ_1 and Δ_2 may be found from statical considerations of Beam *B C* because, disregarding the common elastic factor necessary to change the rotations into end moments, the end shear, V, equals $\frac{M_{CB} - M_{BC}}{l} = \frac{4.5 - 4.2}{l} = \frac{0.3}{l}$, and the moment at the center is $M = 4.2 + \frac{0.3}{l} \times \frac{l}{2} = 4.35$ which is the rotation, Δ_1. To reduce to exact rotations, the flexure factors are divided by $11.2 + 4.2 + 4.35 + 4.35 + 4.5 + 11 = 39.6$. It is to be noted that, for rotations in Span *B C*, the stiffness is doubled since the member is cut in the center. The use of the funicular polygon, although not shown in Fig. 8, may be extended to this case and to the case for a shear influence line.

For shear at Point *B C* (Fig. 8(*f*)), deflect Span *B C* from Span *B A* without rotation and write the flexure factors. They are determined in Span *A B* since $M_{BA} = M_{BC}$. The unit deflection is represented by the quantity, $\frac{2}{3} l \times 11 + \frac{1}{3} l \times 9 = \frac{31}{3} l$. To reduce to exact scale, all the rotations must be divided by $\frac{31}{3} l$.

For a settlement, δ, of Support *B* (Fig. 8(*g*)) the problem is likewise easily solved. The flexure factors are written from the right end, and since $M_{BA} = M_{BC}$, $\Delta_1 = 2\Delta_2$. Equating the deflections and rotations at Point *B* calculated from the right side to those calculated from the left side, $\theta_1 - \frac{1}{3}\Delta_1 = 11 + \frac{2}{3} \times 9 - \frac{1}{3}\Delta_2$ and $-(\theta_1 - \Delta_1) = 11 + 9 - \Delta_2$. Solving these three equations simultaneously, $\Delta_2 = \frac{111}{8}$, $\theta_1 = \frac{173}{8}$, and $\Delta_1 = \frac{111}{4}$. Since δ is represented by $\frac{173}{8} l - \frac{1}{3} l \times \frac{222}{8} = \frac{99\,l}{8}$, the actual rotations are obtained by dividing by $\frac{99\,l}{8\,\delta}\cdot$ Although simultaneous equations were used in the last solution they were very simple and solvable almost by inspection.

The foregoing two problems presented are solved for a very simple case. The solution may be extended, without difficulty, to frames of greater complexity and to members with varying moment of inertia. In general, the directness of the attack made and the inevitable final check furnished by the method of flexure factors will undoubtedly cause it to be viewed with favor as a means of analyzing many problems in continuous structures. The writer does not know of a more direct method for the solution of the complete influence line for a continuous frame.

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DISCUSSIONS

GRIT CHAMBER MODEL TESTS FOR DETROIT, MICHIGAN, SEWAGE TREATMENT PROJECT

Discussion

By G. M. Ridenour, Assoc. M. Am. Soc. C. E.

G. M. Ridenour,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—An interesting approach to a more rational and scientific development of grit chambers is presented in this paper. The similarity to large-scale results in the model, operated in accordance with the model law, is encouraging to further investigation along this line, and the author is to be commended for introducing this method of study.

The difficulty of comparing the effect of depth on the performance of large-scale grit chambers which at the same time vary in so many other respects, such as surface area, volume of flow, distribution of velocities, and ratios of length to width and depth, however, would appear to be great. This seems especially true due to the sensitivity of settling sand to various influences and the exacting relationships of economic calculations.

In view of the seemingly variable conditions between the Grand Rapids, Mich., and Dearborn, Mich., units, the writer wonders whether it is not possible to arrive at conclusions that differ from those presented in the paper, depending on interpretation, and tending to show that the shallower grit chambers might be the more efficient, even at the sacrifice of losing the equality of the theoretical relationship of $\frac{d}{v_s}$ to $\frac{1}{v_h}$ which seems to be the basis for the conclusions in the paper.

From the data presented, the writer has compared the efficiencies of the two units in three different ways: (*a*) On the basis of total percentage of grit removed; (*b*) with respect to the average size of sand removed in succeeding sections of the tank; and (*c*) comparative performance on the basis of equivalent surface areas. The computations are shown in Table 6.

Note.—The paper by George E. Hubbell, Assoc. M. Am. Soc. C. E., was published in December, 1937, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[6] Asst. Prof., Dept. of Water Supplies and Sewage Disposal, Rutgers Univ.; and Research Engr., Div. of Water and Sewage Research, State Agri. Experiment Station, New Brunswick, N. J.

[6a] Received by the Secretary February 3, 1938.

The sieve analysis of the materials removed in each section of the grit chamber shows that, in general, the shallower grit chamber at the Dearborn plant removes a sand of much less average size in equivalent sections of the tank. This action holds throughout the tank length with the exception of the first section which is admittedly adversely influenced by the turbulence of the pumps. Table 6 also shows that the percentage of total sand removed is slightly greater in the Dearborn plant in each succeeding section of the channel, again with the exception of the first 10 ft.

TABLE 6.—COMPARISON OF EFFICIENCIES ON BASIS OF EQUIVALENT SURFACE AREAS

Length	Surface Area			Percentage Removal of Sand, 0.2 to 0.3 Millimeter in Size		Average Size of Sand Removed		Percentage of Total Sand Removed	
	Grand Rapids	Dearborn	Grand Rapids equipment	Grand Rapids	Dearborn	Grand Rapids	Dearborn	Grand Rapids	Dearborn
0–10	40	12.5	86	16.9	11.5	0.73	0.85	37.6	27.2
10–20	120	37.5	256	33.7	56.5	0.47	0.25	21.2	23.7
20–30	200	62.5	428	47.9	89.0	0.43	0.19	11.8	13.6
30–40	280	87.5	600	58.3	110.0	0.52	0.17	5.9	9.5
40–50	360	....	....	65.5	....	0.55	0.16	3.5	3.7
50–60	440	....	....	71.6	....	0.33	0.14	1.9	4.3

Another observation that can be made from the data is the critical size at or below which the percentage of small sand in the composite sample increases toward the outlet end, indicating that point at which the $\frac{1}{v_h}$-ratio exceeds the $\frac{d}{v_s}$-ratio. In the Dearborn plant this size lies between 0.104 and 1.47 mm. In the Grand Rapids plant, the critical size lies between 0.295 and 0.417 mm.

Additional comparisons have been made in Table 6 on the basis of efficiency in relation to equivalent surface area. A sand, 0.2 to 0.3 mm in size, has been used for the calculations, the same as that used for the design of the Detroit plant. These computations seem to confirm the previous generalizations that a greater efficiency can be expected from the shallower unit, provided exception is made of the poor performance in the first 20 ft of the Dearborn chamber due to pumping disturbances. The end of this section is situated at the equivalent surface area of 256 sq ft of the Dearborn unit. The efficiency of the shallower unit is greater beyond this point.

All these observations would seem to indicate a superior efficiency of the shallower Dearborn unit in contrast to Mr. Hubbell's conclusions that a slightly greater removal efficiency can be expected from the deeper tank. Admittedly, the writer has a disadvantage in the interpretation of the results through lack of detailed data on the investigation, but it does seem that on this basis the author's conclusions are not entirely reconcilable with the data presented. If the writer has not erred in interpretation and if Mr.

Hubbell's conclusions are also correct, it then seems that other factors, due to variable conditions in the two units, have played an important part. One such factor which the writer has always considered of importance is the effect of ratios of length to depth on the efficiency of grit chambers. Theoretically, the chamber should be designed so that the value of $\frac{1}{v_h} = \frac{d}{v_s}$ for the smallest particle to be removed. This presupposes, however, that this and all larger particles will settle without turbulence and that the sweeping effect of unequal velocities is not to be present. In actual conditions, this quiescent state does not exist and any given size particle in the liquid is carried farther toward the outlet end of the tank. The distance to which larger sized particles are carried along the tank length is proportionally less, however, than any given decrease in the depth of the tank. The advantage of this fact is gained in a shallower tank. It seems to the writer that the results in Mr. Hubbell's paper also give evidence in this direction.